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A linear programming formulation for autonomous intersection control within a dynamic traffic assignment and connected vehicle environment

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ABSTRACT

This paper develops a novel linear programming formulation for autonomous intersection control (LPAIC) accounting for traffic dynamics within a connected vehicle environment. Firstly, a lane based bi-level optimization model is introduced to propagate traffic flows in the network, accounting for dynamic departure time, dynamic route choice, and autonomous intersection control in the context of system optimum network model. Then the bi-level optimization model is transformed to the linear programming formulation by relaxing the nonlinear constraints with a set of linear inequalities. One special feature of the LPAIC formulation is that the entries of the constraint matrix has only $\{-1,0,1\}$ values. Moreover, it is proved that the constraint matrix is totally unimodular, the optimal solution exists and contains only integer values. It is also shown that the traffic flows from different lanes pass through the conflict points of the intersection safely and there are no holding flows in the solution. Three numerical case studies are conducted to demonstrate the properties and effectiveness of the LPAIC formulation to solve autonomous intersection control.

1. Introduction

Connected vehicle (CV) technology grows rapidly since its inception owing to the development of wireless communication technology, especially the Dedicated Short Range Communications (DSRC) technology. DSRC has great potential in the area of intelligent transportation systems (ITS), as it facilitates the inter-vehicle communication (IVC). The CV technology is the first step toward the next generation transportation system featuring self-driving vehicles, auto highways, auto parking lots, and auto intersection managements. On the one hand, the CV technology greatly improves the transportation system. According to the U.S. Department of Transportation's (DOT) Research and Innovative Technology Administration (RITA, 2014), the CV technology will potentially reduce 81% of all-vehicle target crashes, 83% of all light-vehicle target crashes, and 72% of all heavy-truck target crashes annually. It will also improve the congestion problem in US which consumes up to 4.2 billion hours and 2.8 billion gallons of fuel annually. Due to these benefits, the U.S. DOT National Highway Traffic Safety Administration (NHTSA, 2014) plans to mandate IVC technology on every single vehicle by 2016. On the other hand, the CV technology brings new challenges to the area of traffic control. A key motivation of this study is to address the intersection control problem under the CV environment.

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Notations

Sets

C	set of all lanes
$C_{R,S}$	set of origin, or destination lanes
$C_{O,D,M}$	set of ordinary, or diverging, or merging lanes
E	set of all connectors
$E_{O,D,M}$	set of ordinary, or diverging, or merging lane connectors
E_F	set of conflict lane connectors
$\Gamma^{-1}(i)$	set of predecessors of lane i
$\Gamma(i)$	set of successors of lane i
U	set of all OD pairs
R	set of all paths
R^w	set of paths for OD pair w

Parameters

S	saturation flow rate
T	size of time step
T_f	time horizon for traffic flow modeling
T_d	time horizon for departure time choice. $T_d \leqslant T_f$
N_i^t	jam density of lane i at time t
Q_i^t	inflow or outflow capacity of lane i at time t
W_i	shockwave speed of lane i
V_i	free flow speed of lane i
h_i^r	penalty label of path r at lane i
D_w	total demand of original lane i

Variables

$x_i^{r,t}$	lane occupancy of lane i at time t on path r
$y_{i,j}^{r,t}$	flow from lane i to j at time t on path r
x_i^t	aggregate lane occupancy of lane i at time $t, x_i^t = \sum_{\forall r} x_i^{r,t}$
$y_{i,j}^t$	aggregate flow from lane i to j at time t, $y_{i,j}^t = \sum_{\forall r} y_{i,j}^{r,t}$

1.1. Literature review

Arem van et al. (2006) examined the impact of Cooperative Adaptive Cruise Control under the CV environment and found that both the traffic flow stability and efficiency are improved. However, the study is limited to the cooperation of only connected vehicles. Dresner and Stone (2008), Fajardo et al. (2011), Wuthishuwong and Traechtler (2013) proposed the multi-agent framework for autonomous intersection management (AIM) where vehicles cooperate not only with other vehicles but also with infrastructure (e.g., the intersection controller). In AIM, autonomous vehicles and intersection controller are modeled as intelligent agents. Before reaching the intersection, vehicle agents send requests to signals ahead of the intersection. In consequence, the intersection agent reserves conflict-free trajectories for vehicles to safely pass through the intersection. It is shown that AIM tremendously improves traffic throughout in isolated intersections. Hausknecht et al. (2011) further examined the performance of AIM in the case of multi-intersections and significant improvements were also observed as compared to conventional signal control.

Following this research stream but from a different perspective, Lee and Park (2012) developed the Cooperative Vehicle Intersection Control (CVIC) system. In the framework, autonomous intersection control is formulated as an optimization problem with the objective to minimize the total length of overlapped trajectories. To solve the optimization problem, the active-set algorithm and interior point algorithm are proposed. However, these two algorithms may not necessarily output collision-free solutions due to the complexity of the optimization problem (both the objective function and constraints are nonlinear and non-convex). Accordingly, a Genetic Algorithm based solution is included in the final solution set of the problem. Still the final solution set may contain solutions that cause vehicle trajectory collision. To guarantee the safety of vehicle movements, an accident-free scheme is incorporated in the control strategies of the intersection controller. Later Lee et al. (2013) extended the CVIC framework to the case of multi-intersections and investigated the safety and environmental benefits of autonomous intersection control.

Notice that the traffic flow models in the above literature are all second-order traffic flow models. For example, in (Dresner and Stone, 2008; Fajardo et al., 2011), the acceleration schedule for the driver agent needs to be determined in the modeling component called First-Come-First-Service policy. In (Lee and Park, 2012; Lee et al., 2013), the microscopic