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A bi-level model of dynamic traffic signal control with continuum approximation

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ABSTRACT

This paper proposes a bi-level model for traffic network signal control, which is formulated as a dynamic Stackelberg game and solved as a *mathematical program with equilibrium constraints* (MPEC). The lower-level problem is a *dynamic user equilibrium* (DUE) with embedded *dynamic network loading* (DNL) sub-problem based on the LWR model (Lighthill and Whitham, 1955; Richards, 1956). The upper-level decision variables are (time-varying) signal green splits with the objective of minimizing network-wide travel cost. Unlike most existing literature which mainly use an on-and-off (binary) representation of the signal controls, we employ a continuum signal model recently proposed and analyzed in Han et al. (2014), which aims at describing and predicting the aggregate behavior that exists at signalized intersections without relying on distinct signal phases. Advantages of this continuum signal model include fewer integer variables, less restrictive constraints on the time steps, and higher decision resolution. It simplifies the modeling representation of large-scale urban traffic networks with the benefit of improved computational efficiency in simulation or optimization. We present, for the LWR-based DNL model that explicitly captures vehicle spillback, an in-depth study on the implementation of the continuum signal model, as its approximation accuracy depends on a number of factors and may deteriorate greatly under certain conditions. The proposed MPEC is solved on two test networks with three metaheuristic methods. Parallel computing is employed to significantly accelerate the solution procedure.

1. Introduction

Signalized intersections play a vital role in the design, management and control of urban traffic networks. These locations are often very important restrictive bottlenecks, and therefore urban traffic control strategies tend to focus on the operation of signalized intersections (Miller, 1963; Robertson and Bretherton, 1974; Shelby, 2004; Guler and Cassidy, 2012; Gayah and Daganzo, 2012).

There are multiple approaches of designing and optimizing traffic signal controls on a road network, among which we distinguish between heuristic methods, such as feedback control, genetic algorithms and fuzzy logic; and exact approaches such as mathematical programming and optimal control. Cao et al. (1999) apply fuzzy logic methods to determine useful junction control rules in a dynamic environment. Lin et al. (1997) provide a framework for implementing adaptive traffic

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signal controllers based on fuzzy logic. In another line of research, Improta and Cantarella (1984) formulate a traffic signal control problem at a single road junction as a mixed integer program (MIP). Lo (1999, 2011) employ the cell transmission model (Daganzo, 1994, 1995) to formulate a signal control problem as an MIP. Han et al. (2014) further reduce the number of binary variables and eliminate the vehicle holding problem in the MIP by applying the link transmission model (Yperman et al., 2005). These optimization problems reviewed above are viewed as single-level problems as they treat traffic load at the signalized network as exogenous and do not address drivers' reactions to the implemented signal timings in terms of their route or departure time choices.

On the other hand, signal optimization problems that address drivers' adaptive travel choices as a result of the change in signal timings tend to exhibit a bi-level structure. On the lower level, road users are modeled as non-cooperative Nash players who selfishly minimize their own travel costs by adjusting departure time and/or route choices. On the upper level, a central planner seeks to maximize the network performance through a carefully designed signal plan, which is informed by the prediction of the behavior of the lower-level players (drivers). This bi-level problem is expressed as a *Stackelberg game* and formulated as a *mathematical program with equilibrium constraints* (MPEC).

In a bi-level decision-making environment, the influence of signal timing on route choices is first addressed by Allsop (1974), who considers signal decisions in the presence of a static user equilibrium. Yang and Yagar (1995) apply a sensitivity analysis based on Friesz et al. (1990) to determine the derivatives of link flow and delay in the equilibrium state of a bi-level problem. The above work is extended by Cantarella et al. (1991, 1995), who take into consideration signal settings including cycle length, offset and green split. They derive a lower normative bound and an upper descriptive bound for the solution of signal settings in a traffic network under equilibrium flow. Meneguzzer (1995) develops a route choice user equilibrium (UE) model that incorporates intersection operations with flow-responsive traffic signals. Machemehl and Lee (2005) develop a variety of algorithms such as genetic algorithm, local search optimization and assignment methods to solve an optimal combined UE and signal control problem. Most of the aforementioned studies consider static traffic assignment models and static signal strategies.

In the venue of dynamic traffic modeling and optimization, Abdelfatah and Mahmassani (1998, 2011) consider dynamic traffic signal models and propose a simulation-based solution approach. Gartner and Chronis (1998) present an intelligent transportation system that incorporates both a dynamic traffic assignment module for traffic prediction and a real-time adaptive traffic control system. Chen and Ben-Akiva (1998) consider the combined dynamic user equilibrium (DUE) and signal control problem as a non-cooperative game between traffic controller and road users, which is solved using game-theoretic techniques. Sun et al. (2006) employ a heuristic solution approach for dynamic traffic signal optimization in networks with time-dependent demands and stochastic route choices. The lower level problem in their paper is a *reactive dynamic user optimal* problem, as opposed to the so-called *predictive dynamic user equilibrium* (Friesz et al., 1993). Karoonsoontawong and Waller (2009) propose a mixed-zero-one continuous linear bi-level formulation of the combined dynamic user optimal and traffic signal optimization problem. However, their problem solution is chosen among some pre-defined signal timing plans. Ukkusuri et al. (2013) formulate a signal optimization problem as a game between signal operator and road users. They solve a problem with time-varying signal cycle and split on a small test network with an iterative optimization-and-assignment method. In all the aforementioned dynamic traffic signal control models and methods, the signal controls are captured by binary variables.

This paper presents a bi-level differential Stackelberg game formulation of the network signal control problems with special attention given to the modeling of signalized intersections in the *dynamic network loading* (DNL) sub-problem. In particular, unlike the previously reviewed papers, which all consider the on-and-off (binary) representation of signal controls, we employ the Lighthill-Whitham-Richards (LWR) (Lighthill and Whitham, 1955; Richards, 1956) model integrated with a *continuum signal model* (Han et al., 2014), which does not rely on distinct signal phases. In the continuum signal model, a fraction, η, of the downstream links' capacity is assumed to be available to vehicles discharged from a given approach during the entire signal cycle. η is assumed to be equal to the green proportion of the cycle allocated to the subject approach for movement through the intersection. The continuum signal model has a number of distinctive advantages over its on-and-off counterpart: (1) it requires fewer integer variables when modeling dynamics on large-scale networks, which reduces the computational complexity of the modeling and optimization processes; (2) it provides more flexibility in selecting the time step size and thus increases the computational efficiency; (3) it eliminates discontinuities in path travel times that naturally arise from an on-and-off signal representation, which allows DUE problems to be formulated in a more exact way without the need to introduce user bounded rationality (Szeto and Lo, 2006; Ge and Zhou, 2012; Han et al., 2014); and (4) it allows higher decision resolution in terms of the green time allocated to each approach. The continuum signal model is suitable for predicting the aggregate behavior that exists in large-scale signalized networks without having to handle the detailed and random signal sequences at local intersections. When appropriately utilized, the continuum model can accurately predict the average throughput of traffic bottlenecks, and capture the effect of queue spillbacks.

Nevertheless, as pointed out by Han et al. (2014) through theoretical investigation of the model, the accuracy of the continuum model as an approximation of the on-and-off model is affected by a number of factors, including the fundamental diagram and whether or not spillback is present. Such a complication arises from potential vehicle spillbacks that are captured by the LWR-based DNL model. Following the insights provided by Han et al. (2014), we propose in this paper a DNL procedure that incorporates the continuum signal model and maximizes its approximation efficacy. A numerical case study is provided in Section 3.4 to illustrate the effectiveness of this DNL procedure.