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A cell based dynamic system optimum model with non-holding back flows

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ABSTRACT

This paper proposes a non-holding back linear programming (NHBLP) model with an embedded cell transmission model (CTM), to account for the system optimum dynamic traffic assignment.

1. Introduction

System optimum dynamic traffic assignment (DTA) models have attracted lots of attentions from traffic engineers and researchers in the past two decades. A handful of approaches have been developed to address the problem depending on the underlying network loading models: (1) exit flow function models (Merchant and Nemhauser, 1978a, 1978b; Wie et al., 1995; Nie, 2011); (2) cell-based type of models (Daganzo, 1994, 1995; Ziliaskopoulos, 2000; Waller and Ziliaskopoulos, 2006); (3) store-and-forward models (D'Ans and Gazis, 1976; Papageorgiou, 1990, 1995; Aboudolas et al., 2009). As one of the widely used network loading models, the cell transmission model (CTM) by Daganzo (1994, 1995) has seen popularity for various dynamic problems (Lo, 1999; Lo and Szeto, 2002; Gomes and Horowitz, 2006; Gomes et al., 2008; Ukkusuri et al., 2010; Han et al., 2011; Zhang et al., 2013). To the authors' best knowledge, Ziliaskopoulos (2000) was the first to incorporate CTM into the SODTA problem for a single destination network. The benefit of the formulation lies in the linear nature of the model, which makes the formulation computationally efficient and solvable for a reasonable size network. However, as noted by several researchers earlier (Ziliaskopoulos, 2000; Shen et al., 2007; Zheng and Chiu, 2011; Nie, 2011; Doan and Ukkusuri, 2012), an important issue with the formulation is the holding-back phenomenon due to the linear relaxation of nonlinear constraints, where traffic flows may be reluctant to move forward to the downstream cells, thus compromising the nonlinear exit flow constraint (more details are elaborated in the later section). Other two limitations of this model are: (1) the formulation only applies to single-destination network; (2) the traffic demand must be able to exit the network at the end of the assignment period.

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Notation

Sets

C	set of all the cells
C_R	set of origin cells
C_S	set of destination cells
C_O	set of ordinary cells
C_D	set of diverging cells
C_M	set of merging cells
E	set of all the cell connectors
E_O	set of ordinary cell connectors
E_D	set of diverging cell connectors
E_M	set of merging cell connectors
$\Gamma^{-1}(i)$	set of predecessors of cell i
$\Gamma(i)$	set of successors of cell i

Parameters

S	saturation flow rate
T_f	time horizon
N_i^t	jam density of cell i at time t
Q_i^t	inflow or outflow capacity of cell i at time t
δ_i	ratio of free flow speed over shockwave speed at cell i, within $[0,1]$
h_i	penalty label of cell i
d_i^t	demand of cell i at time t
M_S	a small positive constant
M_L	a large positive constant

Variables

x_i^t	aggregate cell occupancy of cell i at time t
$y_{i,j}^t$	aggregate flow from cell i to j at time t
m	dummy variable to remove holding-back flows

1.1. Literature review of the holding back problem

The holding-back problem has been a well-known problem for SO models in the literature (Carey and Subrahmanian, 2000; Peeta and Ziliaskopoulos, 2001). Lin and Wang (2004) addressed the problem of holding-back by introducing an additional term, a sufficiently small coefficient multiplying the sum of outflow travel time, to the objective function. However, there is no guarantee that such term always removes holding flows. The authors did not discuss the determination of the coefficient or show any proof that holding flows are eliminated. Pavlis and Recker (2009) described the nonlinear minimum constraints of flow as "if–then" implications and transformed the conditional piecewise functions into a set of mixed integer linear inequality constraints. The holding-back problem is handled in an explicit manner but the difficulty lies in solving the mixed integer problem. Shen et al. (2007) proved that optimal solution with non-holding flows will always exist in a simplified SODTA problem and presented the simplex method to solve the formulation. However, the traffic flow model applied in the simplified formulation is a point queue model, rendering the formulation incapable of capturing shockwave and spill back phenomenon in traffic propagation. More recently, Nie (2011) showed that CTM can be derived as a special case from the original M–N model (Merchant and Nemhauser, 1978a, 1978b). Moreover, Nie verified that Ho's algorithm (Ho, 1980) can be utilized to eliminate the unnecessary holding flows in the network. However, it cannot guarantee all the holding-back flows can be removed depending on the magnitude of assignment horizon.

From a different perspective, Zheng and Chiu (2011) explored the cell-based SODTA problem in the path flow level rather than the link flow level, and proved that the original nonlinear SODTA is equivalent to the earliest arrival flow (EAF) problem, specifically for the single destination network. They proposed an augmenting path algorithm to obtain solutions without holding flow in ordinary and merging cell connectors. However, the EAF may not hold for non-single destination network, thus the scope of the algorithm is limited. Most recently, Doan and Ukkusuri (2012) utilized the concept of fair propagation and proposed a novel formulation that completely eliminates the holding-back phenomenon for networks with multiple OD pairs. Still, the limitation lies in maintaining the non-linear form of constraint sets, thus increasing the difficulty of finding a non-holding flow solution. Moreover, the concept of fair propagation might not necessarily reflect the realistic traffic flow characteristics.