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Surface reconstruction from unorganized points with l_0 gradient minimization

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ABSTRACT

To reconstruct surface from unorganized points in three-dimensional Euclidean space, we propose a novel efficient and fast method by using l_0 gradient minimization, which can directly measure the sparsity of a solution and produce sharper surfaces. Therefore, the proposed method is particularly effective for sharpening major edges and removing noise. Unlike the Poisson surface reconstruction approach and its extensions, our method does not depend on the accurate directions of normal vectors of the unorganized points. The resulting algorithm is developed using a half-quadratic splitting method and is based on decoupled iterations that are alternating over a smoothing step realized by a Poisson approach and an edge-preserving step through an optimization formulation. This iterative algorithm is easy to implement. Various tests are presented to demonstrate that our method is robust to point noise, normal noise and data holes, and thus produces good surface reconstruction results.

1. Introduction

Given a set of unorganized three-dimensional (3D) points, the purpose of surface reconstruction is to restore the surface of the original object where the points are scanned. The industry of 3D printing has emerged in the past few years, thus 3D surface reconstruction to visualize objects in space has become one of the main issues in the fields of both applied mathematics (Mancosu et al., 2005) and computer science (Bellocchio et al., 2013). In the past few decades, many algorithms have been developed to solve the surface reconstruction problem (Avron et al., 2010; Berger et al., 2017; Bödis-Szomorü et al., 2017; Calakli and Taubin, 2011; Carlini and Ferretti, 2017; Huang et al., 2009; Kazhdan et al., 2006; Kazhdan and Hoppe, 2013; Khatamian and Arabnia, 2016; Kolluri et al., 2004; Li and Kim, 2015; Li et al., 2014; Lipman et al., 2007; Liu and Wang, 2012; Liu et al., 2016; Ohtake et al., 2005; Reinhold et al., 2014; Xiong et al., 2014; Zagorchev and Goshtasby, 2012; Zhao et al., 2001, 1998). However, it is still a very challenging task due to the missing information of point orders, orientations, connections, as well as complex surface topologies. In general, existing surface reconstruction techniques can be classified into two types: explicit mesh-based reconstruction and implicit volume-based reconstruction.

The explicit mesh-based reconstruction techniques use the unorganized points directly to form a triangular mesh. Kolluri et al. (2004) introduced a noise-resistant method for reconstructing a watertight surface from point cloud data. Lipman et al. (2007) presented a parameterization-free projection algorithm that does not need the local parameters. This approach was extended in Huang et al. (2009) and Reinhold et al. (2014). Xiong et al. (2014) proposed a unified method that treats connectivity construction and geometry as one joint optimization problem. Avron et al. (2010) presented an l_1-sparse approach for reconstruction of point set surfaces with sharp features. All these mesh-based reconstruction schemes are precise, but they have difficulties in dealing with noise, complex topologies, and especially holes in data.

The implicit volume-based reconstruction approaches generally construct an implicit volume-function from the input points, and then obtain the restored surface from the iso-surface of the volume-function. Liu et al. (2016) and Liu and Wang (2012) introduced a method to fit the points with radial basis functions, then the surface is defined as the zero level-set of those radial basis functions. Carlini and Ferretti (2017) proposed a semi-Lagrangian method coupled with radial basis function interpolation for computing a curvature-related level set model. Ohtake et al. (2005) suggested an implicit surface representation using

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multi-level partition of a unity. The local shape was approximated by a weighted piecewise quadratic function. Zhao et al. (2001, 1998) proposed the construction of a stopping function that acts to stop the evolution when the contour reaches the surface data points. The main advantage of these methods is that it can easily reconstruct surface with complex topologies, and can also be implemented concisely. A curvature-adaptive implicit surface reconstruction for irregularly spaced points in 3D space was introduced in Zagorchev and Goshtasby (2012). One of the best-known techniques is Poisson surface reconstruction Kazhdan et al. (2006), in which the implicit function is used as the indicator function of the volume bounded by the surface. This function is obtained by solving a Poisson equation and is identically equal to one inside, zero outside, and discontinuous on the reconstructed surface. The main disadvantage of this method was improved by Kazhdan and Hoppe (2013) adding constraints on the points to avoid over-smoothing of the reconstructed surface. Furthermore, Calakli and Taubin (2011) suggested adding a higher-order regularization term and introduced the Hessian matrix of the indicator function. Recently, we presented a novel fast and accurate phase field model for surface embedding narrow volume reconstruction from an unorganized surface data set. The methods proposed in Li et al. (2014) and Li and Kim (2015) were based on the Allen–Cahn (AC) equation (Allen and Cahn, 1979), which has the motion by the surface mean curvature and can be applied to image processing problems (Li and Kim, 2011, 2012). We choose the AC equation because an accurate and fast hybrid numerical solver is available (Li et al., 2010). The phase field model can be directly used to reconstruct a surface from the point cloud.

All these implicit methods used the l_1- or l_2-norm in their proposed minimization or energy term. However, sometimes they suffer from a tendency to oversmooth the data. To produce sharper surfaces than either the l_1- or l_2-norm, in this paper, we present a novel accurate and fast method by using l_0 gradient minimization, which can directly measure the sparsity of a solution and produce a sharper surface. Therefore, the proposed method is particularly effective for sharpening major edges and removing noise. To the best of the authors' knowledge, there are no existing methods for surface reconstruction using the l_0-norm, which can produce the sparsest solutions. Unlike the Poisson surface reconstruction approach and its extended approaches, our method does not depend on accurate directions of normal vectors of the unorganized points. The resulting algorithm is developed by using a half-quadratic splitting method (He et al., 2014) and is based on decoupled iterations that are alternating over a smoothing step by a Poisson approach and an edge-preserving step using an optimization formulation. This iterative technique is fast, simple, and easy to implement. Various numerical tests are presented to demonstrate that our method is robust to point noise, normal noise, and data holes, and thus produces good surface reconstruction results.

Our paper is organized as follows. In Section 2, the proposed method for surface reconstruction is given. We describe the proposed optimization method in Section 3. In Section 4, experimental results and comparisons are given. We draw the conclusions in Section 5. In Appendix B, we present the numerical solver.

2. Description of the proposed model

The implicit method uses the data set to define a signed distance function on Cartesian grids and the reconstructed surface is defined as the zero iso-surface of the signed distance function. Let us briefly review the definition of the signed distance function. At a point $\mathbf{x}$ in the domain Ω, we denote by $\overline{\mathbf{X}}$ the data point that is closest to $\mathbf{x}$. Then, we define the signed distance function as

$$d(\mathbf{x}) = s(\mathbf{x})\bar{d}(\mathbf{x}). \tag{1}$$

Here $\bar{d}(\mathbf{x})$ is the unsigned distance function, which is defined as $\bar{d}(\mathbf{x}) = ||\mathbf{x} - \overline{\mathbf{X}}||$ and $s(\mathbf{x})$ is the sign of the signed distance function $d(\mathbf{x})$, which is defined as

$$s(\mathbf{x}) = \text{sign}((\mathbf{x} - \overline{\mathbf{X}})\cdot\mathbf{N}(\overline{\mathbf{X}})). \tag{2}$$

Here, sign() is the sign function, which is defined as 1 or − 1 for positive or for negative arguments, respectively and $\mathbf{N}$ is the outward normal vector at the surface point. The inner product field is defined as the signed distance of a grid point to the tangent plane of the closest surface point. However, even the normal vectors are given with high accuracy, if the vectors $\mathbf{x} - \overline{\mathbf{X}}$ and $\mathbf{N}(\overline{\mathbf{X}})$ are nearly perpendicular, i.e., $(\mathbf{x} - \overline{\mathbf{X}})\cdot\mathbf{N}(\overline{\mathbf{X}}) \approx 0$, or $\mathbf{N}(\overline{\mathbf{X}})$ is with noise, $s(\mathbf{x})$ will be with noise. In practice, there are outliers or conflicting points in an unorganized point cloud. The normal vectors for the point cloud may be absent in practice. However, the direction vectors exist when the points are scanned from a 3D machine. Note that the normal information is not necessary for our proposed method. Although $s(\mathbf{x})$ is with noise, the numerical tests in Section 4 indicate that the proposed method can accurately remove noise and produce good results. Let us consider the following l_0 gradient regularization version:

$$\min_{\phi} \int_{\Omega} g(\mathbf{x}) \; ||\nabla\phi||_0 \, d\mathbf{x}, \tag{3}$$

where

$$g(\mathbf{x}) = \tanh(\bar{d}(\mathbf{x})/(\sqrt{2}\xi)). \tag{4}$$

Here, ξ is related to the interface transition thickness. $g(\mathbf{x})$ is a weighted function (see Fig. 1(b) and (d)), which is almost zero near the data set and is non-negative in the other regions. The gradient $\nabla\phi(\mathbf{x}) = (\partial_x\phi, \partial_y\phi, \partial_z\phi)$ for each grid $\mathbf{x}$ denotes the vector differential operator along the x-, y-, and z-directions. The l_0-norm of a vector $\nabla\phi$, i.e., $||\nabla\phi||_0 = |\partial_x\phi|^0 + |\partial_y\phi|^0 + |\partial_z\phi|^0$, which directly measures the sparsity and enforces the surface to be sharper. Here, we define $0^0 = 0$. The initial surface $\phi^0(\mathbf{x})$ (see Fig. 1(c) and (d)) is chosen as

$$\phi^0(\mathbf{x}) = \tanh(d(\mathbf{x})/(\sqrt{2}\xi)), \tag{5}$$

which is defined by $\phi^0(\mathbf{x}) \approx 1$ in the interior region and $\phi^0(\mathbf{x}) \approx -1$ in the exterior region. The reconstructed surface is defined by $\phi^0(\mathbf{x}) = 0$. The initial surface is usually not bad because most of the given data points are located at the reconstructed surface. Note that our approach does not require all exterior (interior) grid points to be identified correctly. We identify as many correct exterior grid points as possible because a good initial implicit surface can reduce the computational cost significantly.

The following points should be noted:

(1) Since the function $g(\mathbf{x})$ is almost zero near the data set and is non-negative in the other regions, $g(\mathbf{x})$ also acts to stop the evolution when the contour reaches the surface data points.
(2) The l_0-norm minimization for $\nabla\phi(\mathbf{x})$ can produce sharper surface edges than those of either l_1-norm or l_2-norm.
(3) Combining the above two advantages, the reconstructed surface is close to the original point clouds and has sharper surfaces with high quality.

We need to define the value of the function along the domain boundary for Eq. (3). If we assume that the reconstructed surface is away from the boundary of the domain, we can use the Dirichlet boundary condition, $\phi = -1$. Meanwhile, the zero Neumann boundary condition enforces that the normal derivative is zero along the boundary. A periodic boundary condition can also be used because the triply periodic minimal surfaces exist (Li and Guo, 2017; Li et al., 2016; Torquato and Donev, 2004).

3. Optimization of the proposed model

The minimization problem (3) is difficult to optimize due to the