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Ultimate levelings

Wonder A.L. Alves[a,b], Ronaldo F. Hashimoto[b,*], Beatriz Marcotegui[c]

[a] Informatics and Knowledge Management Graduate Program, Universidade Nove de Julho, São Paulo, Brazil
[b] Department of Computer Science, Institute of Mathematics and Statistics, Universidade de São Paulo, São Paulo, Brazil
[c] Centre for Mathematical Morphology, Mines-ParisTech, Fontainebleau, France

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ABSTRACT

This work presents a new class of residual operators called ultimate levelings which are powerful image operators based on numerical residues. Within a multi-scale framework, these operators analyze a given image under a series of levelings. Thus, contrasted objects can be detected if a relevant residue is generated when they are filtered out by one of these levelings. Our approach consists of, firstly, (*i*) representing the input image as a morphological tree; then, (*ii*) showing that a certain operation on this tree results in a leveling operator; and finally (*iii*) demonstrating that a sequential application of this operation on the tree is able to produce a family of levelings that satisfies scale-space properties. Besides, other contributions of this paper include: (*i*) the statement of properties of ultimate levelings, (*ii*) the presentation of an efficient algorithm for their computation, (*iii*) the provision of strategies for choosing families of primitives, (*iv*) the presentation of strategies for filtering undesirable residues, and (*v*) the provision of some illustrative examples of application of ultimate levelings. Furthermore, ultimate levelings are computationally efficient and their performance evaluations are comparable to the state of art methods for filtering and image segmentation.

1. Introduction

An operator in Mathematical Morphology (MM) can be seen as a mapping between complete lattices (Serra, 1988). In particular, mappings on the set of all gray level images $\mathcal{F}(\mathcal{D})$ defined on domain $\mathcal{D} \subset \mathbb{Z}^2$ and co-domain $\mathbb{K} = \{0, 1, \ldots, K\}$ are of special interest in MM, i.e., $\psi : \mathcal{F}(\mathcal{D}) \to \mathcal{F}(\mathcal{D})$. Furthermore, when ψ satisfies the properties of being increasing ($\forall f, g \in \mathcal{F}(\mathcal{D}), f \leq g \iff \psi(f) \leq \psi(g)$) and idempotent ($\forall f \in \mathcal{F}(\mathcal{D}), \psi(f) = \psi(\psi(f))$), it is called *morphological filter* (Serra, 1988). The first condition preserves the lattice relation order, while the second condition responds to the fact that increasing operations are not invertible. By using these properties, information content reductions are expected after applying morphological filters (Serra and Vincent, 1992). Relying on these characteristics, morphological filters remove selectively undesirable contents from images such as noise, background irregularities, etc.; while preserving desired contents (Bangham et al., 1996a, 1996b; Najman and Talbot, 2013). However, this is not always an easy task.

A complementary strategy is to effectively erase the desirable portion of an image, and then restore it through a difference with the original image. This gives rise to the idea of *residual operators*. Simply put, residual operators are transformations that involve combinations of morphological operators with differences (or subtractions). Morphological gradient, top-hat transforms, skeleton by maximal balls, ultimate erosion and ultimate opening are some examples of residual operators widely used in image processing applications.

In this study, we present a new large class of residual operators based on an indexed family of levelings. This class of residual operators analyzes the evolution of the residual value between two consecutive operators on a scale-space of levelings. The residual value of these operators can reveal important contrast information in images. This new class of operators includes some existing ones such as maximum difference of openings (resp., closings) by reconstruction (Li et al., 1997), differential morphological profiles (Pesaresi and Benediktsson, 2000), ultimate attribute openings (resp., closings) (Retornaz and Marcotegui, 2007), differential attribute profiles (Dalla Mura et al., 2010), shape ultimate attribute openings (resp., closings) (Hernández and Marcotegui, 2011) and ultimate grain filters (Alves and Hashimoto, 2014). They have successfully been used as a preprocessing step in various applications such as texture features extraction (Li et al., 1997), segmentation of high-resolution satellite imagery (Dalla Mura et al., 2010; Pesaresi and Benediktsson, 2001), text location (Alves and Hashimoto, 2010; Retornaz and Marcotegui, 2007), segmentation of building

* Corresponding author.
E-mail addresses: wonder@uni9.pro.br (W.A.L. Alves), ronaldo@ime.usp.br (R.F. Hashimoto), beatriz.marcotegui@mines-paristech.fr (B. Marcotegui).

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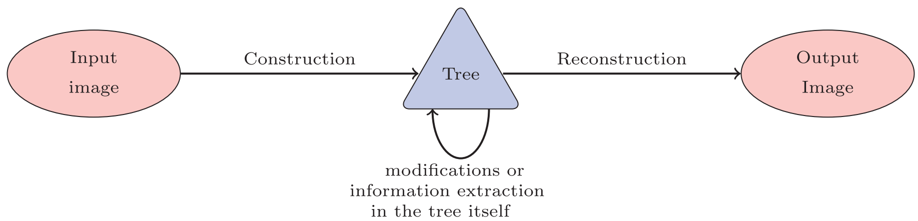

Fig. 1. Image representation through a tree.

façades (Hernández and Marcotegui, 2011) and restoration of historical documents (Meyer, 2010). Given above considerations, in this paper, besides the presentation of this new class of residual operators, which we call *ultimate levelings*, we also show its properties, fast algorithms for its computation and possible applications for residual information extraction in image processing applications.

The remainder of this paper is structured as follows. Section 2 briefly recall some definitions and properties about scale-space representations based on levelings and residual operators. In Section 3, we provide the main contributions of this paper by (*i*) introducing the ultimate leveling as a class of residual operators, (*ii*) stating some of its properties and (*iii*) presenting an efficient algorithm for its computation. Section 4 provides some strategies for choosing a family of primitives. In Section 5, we present some strategies for filtering undesirable residues and, Section 6 provides some illustrative examples of application of ultimate levelings. Finally, Section 7 concludes this work and presents some future research directions.

2. Theoretical background

In many image processing problems, objects of interest may be present at different scales. For such situations, multi-scale approaches have been developed over the last few decades, where a sequence of coarser and coarser decompositions of the same image are derived. In this sense, Meyer and Maragos (2000) proposed a nonlinear scale-space (sequence of images) decomposition obtained from a sequence of operators of a general class of morphological filters called levelings giving rise to *morphological scale-space based on levelings*. Later, Alves et al. (2015) proved that there exists an efficient representation of morphological scale-space based on levelings through hierarchies of level sets.

Thus, for the sake of understanding, Sections 2.1 and 2.2 recall some definitions and properties of hierarchies of level sets and morphological scale-space based on levelings, respectively. Then, we present an efficient representation of morphological scale-spaces based on levelings through hierarchies of level sets. And, Section 2.3 provides a link between the state of art in residual operators and the main result of this paper, given in Section 3.

2.1. Hierarchies of level sets: component tree and tree of shapes

Image representations through trees have been proposed in recent years to carry out tasks of image processing and analysis such as filtering, segmentation, pattern recognition, contrast extraction, registration, compression and others. In this scenario, as illustrated in Fig. 1, the first step consists of constructing a representation of the input image by means of a tree, then the task of image processing or analysis is performed through modifications or information extraction in the tree itself, and finally an image is reconstructed from the modified tree.

In order to build the trees considered in this paper, we need the following definitions. For any $\lambda \in \mathbb{K} = \{0, 1, \dots, K\}$, we define $\mathcal{X}_{\downarrow}^{\lambda}(f) = \{p \in \mathcal{D} : f(p) < \lambda\}$ and $\mathcal{X}_{\lambda}^{\uparrow}(f) = \{p \in \mathcal{D} : f(p) \geq \lambda\}$ as the *lower* and *upper level sets* at value λ from an image $f \in \mathcal{F}(\mathcal{D})$, respectively. These level sets are nested, i.e., $\mathcal{X}_{\downarrow}^{1}(f) \subseteq \mathcal{X}_{\downarrow}^{2}(f) \subseteq \dots \subseteq \mathcal{X}_{\downarrow}^{K}(f)$ and $\mathcal{X}_{K}^{\uparrow}(f) \subseteq \mathcal{X}_{K-1}^{\uparrow}(f) \subseteq \dots \subseteq \mathcal{X}_{0}^{\uparrow}(f)$. It is possible to show that the image f can be reconstructed using either the family of lower or upper sets, i.e., $\forall p \in \mathcal{D}$,

$$f(p) = \sup\{\lambda : p \in \mathcal{X}_{\lambda}^{\uparrow}(f)\} = \inf\{\lambda - 1 : p \in \mathcal{X}_{\downarrow}^{\lambda}(f)\}. \quad (1)$$

Examples of lower and upper level sets are presented in Fig. 2. Observe that, given a $\lambda \in \mathbb{K}$, the level set $\mathcal{X}_{\downarrow}^{\lambda}(f)$ or $\mathcal{X}_{\lambda}^{\uparrow}(f)$ may have more than one connected component.

From lower or upper level sets, we define two other sets $\mathcal{L}(f)$ and $\mathcal{U}(f)$ composed by the connected components (CCs) of the lower and upper level sets of f, i.e., $\mathcal{L}(f) = \{C \in \mathcal{CC}_4(\mathcal{X}_{\downarrow}^{\lambda}(f)) : \lambda \in \mathbb{K}\}$ and $\mathcal{U}(f) = \{C \in \mathcal{CC}_8(\mathcal{X}_{\lambda}^{\uparrow}(f)) : \lambda \in \mathbb{K}\}$, where $\mathcal{CC}_4(X)$ and $\mathcal{CC}_8(X)$ are sets of 4 and 8 connected CCs of X, respectively. The ordered pairs consisting of the CCs of the lower and upper level sets and the usual inclusion set relation, i.e., $(\mathcal{L}(f), \subseteq)$ and $(\mathcal{U}(f), \subseteq)$, induce two dual trees (Caselles et al., 2008) called *component trees*. This leads us to Definition 2.1, and consequently to Proposition 2.2. In Fig. 3, we have examples of component trees $(\mathcal{L}(f), \subseteq)$ and $(\mathcal{U}(f), \subseteq)$ built from the lower and upper level sets presented in Fig. 2. Observe that the CCs of the lower or upper level sets extracted from an image have a hierarchical structure in these trees.

Definition 2.1. Let $(\mathcal{T}, \preceq)$ be an ordered set. We say that $\preceq$ induces a tree structure in $\mathcal{T}$ if the following two conditions hold:

1. $\exists R \in \mathcal{T}$ such that $\forall N \in \mathcal{T}, N \preceq R$. In that case we shall say that R is the root of the tree.
2. $\forall A, B, C \in \mathcal{T}$, if $A \preceq B$ and $A \preceq C$ then either $B \preceq C$ or $C \preceq B$. In that case, we shall say that B and C are nested.

Proposition 2.2. (Caselles et al.(2008)) *Both* $(\mathcal{L}(f), \subseteq)$ *and* $(\mathcal{U}(f), \subseteq)$ *are trees.*

2.1.1. Tree of shapes

Combining these dual trees $(\mathcal{L}(f), \subseteq)$ and $(\mathcal{U}(f), \subseteq)$ into a single tree, the so called *tree of shapes* can be built (Caselles et al., 2008; Monasse and Guichard, 2000). In fact, let $\mathcal{P}(\mathcal{D})$ denote the *powerset* of $\mathcal{D}$ and let $sat : \mathcal{P}(\mathcal{D}) \to \mathcal{P}(\mathcal{D})$ be the operator of saturation (Caselles et al., 2008; Caselles and Monasse, 2010; Monasse and Guichard, 2000). Thus, let $\mathcal{SAT}(f) = \{sat(C) : C \in \mathcal{L}(f) \cup \mathcal{U}(f)\}$ be the family of CCs of the lower and upper level sets, respectively, with all holes filled. The elements of $\mathcal{SAT}(f)$, called *shapes*, are nested by the inclusion relation. The pair $(\mathcal{SAT}(f), \subseteq)$ induces a tree (see Proposition 2.3) which is called *tree of shapes* (Caselles et al., 2008; Caselles and Monasse, 2010). Fig. 3, we have an example of tree of shapes $(\mathcal{SAT}(f), \subseteq)$ built from the lower and upper level sets presented in Fig. 2.

Proposition 2.3. (Caselles et al.(2008)) *The ordered set* $(\mathcal{SAT}(f), \subseteq)$ *is a tree.*

2.1.2. Compact representation of trees

Tree of shapes and also component trees of an image f will be denoted generically by $\mathcal{T}_f$. It is well known that a tree $\mathcal{T}_f$

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