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Distributed fusion filter for multi-sensor systems with finite-step correlated noises

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ABSTRACT

This paper addresses the distributed fusion filtering problem for multi-sensor systems with finite-step correlated noises. The process noise and observation noises of different sensors are finite-step auto- and cross-correlated, respectively. Based on the optimal local filtering algorithms that we presented before, the filtering error cross-covariance matrices between any two local filters are derived based on an innovation analysis approach. A distributed fusion filter is put forward by using matrix-weighted fusion estimation algorithm in the linear unbiased minimum variance sense. Finally, the proposed algorithms are extended to systems with random parameter matrices. Two simulation examples are given to show the effectiveness of the proposed algorithms.

1. Introduction

In the past few decades, in order to meet the needs of higher accuracy, various sensors have been used in many practical systems since they can provide more information on the target in time and space. Information fusion estimation for multi-sensor systems has attached much attention due to the wide applications in engineering systems such as target tracking, environment monitoring, fault diagnosis, to name a few [1,2].

Classically, centralized and distributed fusion are generally used to process the information from multiple sensors [2]. In the centralized fusion estimation, all raw observation data are sent to a fusion center. Therefore, the algorithm can provide globally optimal estimate when there are no faulty sensors. Its disadvantage is that it is not robust and less flexible when there are faulty sensors. To avoid the shortcoming, the distributed fusion is proposed. Information from each sensor is preprocessed to produce a local estimate, and then all local estimates are sent to a fusion center for producing a fusion estimate by certain fusion criterion in distributed fusion. The distributed fusion estimator derived is globally suboptimal in general since there are information losses. The great virtue of distributed fusion is the robustness and flexibility brought by its parallel structure which makes fault detection and isolation of sensors easy [3–6]. Thus, the distributed processing is preferred by many practical applications [7], and see [8,9] for recent surveys.

The estimation problems for networked control systems (NCSs) have been highlighted and widely investigated because of its advantages such as flexibility, robustness, low cost and so on [10,11]. It is linked by networks among sensors, estimators, controllers and actuators in NCSs. Thus, the uncertainties of random delays and packet dropouts are inevitable because of the unreliability of transmission channels. Networked systems with random delays and packet dropouts can be transformed to random parameterized systems with correlated noises. The uncertainties of random parameters can be described by multiplicative noises. The estimation problems for stochastic uncertain systems have become research focuses in recent years [12–19].

Correlated noises usually exist in the practical applications: e.g., a discretized system from a continuous-time system, a system in a common noisy resource, a system with multiple measurement delays, reduced-order subsystems from a stochastic singular system, and so on. They are subject to correlated noises which will deteriorate the performance of a system. The state estimation problems of uncertain systems with correlated noises need discussing in depth. Many valid methods are available in recent literature. For a class of systems with correlated noises, a linear optimal recursive filter is proposed in [20] with applications to networked systems with one-step random delay and missing measurements. For systems with packet dropouts and finite-step auto-correlated noises, an optimal filter [21] and a suboptimal Kalman-type filter [22] have been designed. Optimal linear estimators are developed for systems with finite-step correlated noises and packet dropout compensation in [23]. Furthermore, optimal linear estimators including filter, predictor and smoother [24] and a suboptimal Kalman-type filter [25] have also been proposed for systems with random parameter matrices, multiple fading measurements, stochastic nonlinearities, and finite-step auto- and cross-correlated noises. In the literature aforementioned, the optimal or suboptimal estimators are only

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investigated for single sensor systems with correlated noises, but the fusion estimation problem for multi-sensor systems with correlated noises is not taken into account. For multi-sensor uncertain systems with correlated noises, a distributed weighted Kalman filter is proposed based on the suboptimal Kalman-type local filter in [26], and centralized fusion estimators and distributed fusion filter are presented in [27]. However, those fusion estimators are only designed for systems with one-step auto-correlated and/or two-step cross-correlated noises. Thus far, distributed fusion filter for multi-sensor systems with finite multi-step auto- and cross-correlated noises has not yet been presented because the derivation of cross-covariance matrices between any two local filters is difficult and has not solved yet.

In view of the above considerations, the objective of this paper is to solve the distributed fusion filtering algorithm for multi-sensor systems with finite-step auto- and cross-correlated noises. Based on the local filtering algorithm of our previous work [23], the filtering error cross-covariance matrices between any two local filters are deduced via an innovation analysis approach for the first time. Based on local filters and the cross-covariance matrices, a distributed fusion filter is obtained by using matrix-weighted fusion algorithm in the linear unbiased minimum variance sense [6]. At last, the proposed algorithms are generalized to solve the systems with random parameter matrices which have wide applications in networked systems with random delays and/or packet dropouts [13–19,28–32].

The outlines of this paper are summarized as follows. Section 2 gives the problem formulation. The optimal local filter is introduced in Section 3. The filtering error cross-covariance matrices of any two local filters are deduced in Section 4. In Section 5, the distributed fusion filter weighted by matrices is given. In Section 6, the proposed algorithms are generalized to solve a random parameterized system with finite-step correlated noises. In Section 7, two examples are given to show the effectiveness of the proposed algorithms. The last part of this paper is the conclusion. Appendixes give the proofs of main results.

Notation: The notation used here is standard. R^n denotes the n-dimensional Euclidean space; Superscript T denotes the transpose; E denotes the mathematical expectation; Cov denotes the covariance; δ_{tk} is the Kronecker delta function; I_{p_i} is a p_i by p_i identity matrix; $\perp$ denotes orthogonality. $\hat{x}(\circ|\bullet)$ denotes the estimate of stochastic variable $x(\bigcirc)$ based on information before time $\bullet$, i.e., the projection of $x(\bigcirc)$ on the linear space generated by information before time $\bullet$. $\tilde{x}(\circ|\bullet) = x(\circ) - \hat{x}(\circ|\bullet)$ denotes the estimation error. $P_{\tilde{x}\tilde{y}}(\circ, *|\blacklozenge, \bullet) = \mathrm{E}[\tilde{x}(\circ|\blacklozenge)\tilde{y}^{\mathrm{T}}(*|\bullet)]$ is the covariance matrix between estimation errors $\tilde{x}(\circ|\blacklozenge)$ and $\tilde{y}(*|\bullet)$, with $P_{\tilde{x}\tilde{x}}(\circ, *|\blacklozenge, \bullet) = P_{\tilde{x}}(\circ, *|\blacklozenge, \bullet)$, $P_{\tilde{x}\tilde{y}}(\circ, \circ|\blacklozenge, \bullet) = P_{\tilde{x}\tilde{y}}(\circ|\blacklozenge, \bullet)$, $P_{\tilde{x}\tilde{y}}(\circ, *|\bullet, \bullet) = P_{\tilde{x}\tilde{y}}(\circ, *|\bullet)$ and $P_{\tilde{x}\tilde{y}}(\circ, \circ|\bullet, \bullet) = P_{\tilde{x}\tilde{y}}(\circ|\bullet)$, and $\mathrm{E}[\tilde{x}(\circ|\bullet)\tilde{y}^{\mathrm{T}}(*|\bullet)] = \mathrm{E}[x(\circ)\tilde{y}^{\mathrm{T}}(*|\bullet)] = \mathrm{E}[\tilde{x}(\circ|\bullet)y^{\mathrm{T}}(*)]$ with the definitions $P_{x\tilde{y}}(\circ, *|\bullet) = \mathrm{E}[x(\circ)\tilde{y}^{\mathrm{T}}(*|\bullet)]$ and $P_{\tilde{x}y}(\circ, *|\bullet) = \mathrm{E}[\tilde{x}(\circ|\bullet)y^{\mathrm{T}}(*)]$. $Q_{xy}(\circ, *) = \mathrm{E}[x(\circ)y^{\mathrm{T}}(*)]$ is the second-order moment matrix between variables $x(\bigcirc)$ and $y(*)$, with $Q_{xx}(\circ, *) = Q_x(\circ, *)$ and $Q_{xy}(\circ, \circ) = Q_{xy}(\circ)$.

2. Problem formulation

Consider the following stochastic system with finite-step correlated noises:

$$x(t+1) = \Phi(t)x(t) + \Gamma(t)w(t) \tag{1}$$

$$y_i(t) = H_i(t)x(t) + v_i(t), \quad i = 1, \cdots, L \tag{2}$$

where $x(t) \in R^n$ is the system state to be estimated. $y_i(t) \in R^{p_i}$, $i = 1, 2, \cdots, L$ is the observation of the ith sensor. $w(t) \in R^m$ and $v_i(t) \in R^{p_i}$ are the process and observation noises. $\Phi(t)$, $\Gamma(t)$ and $H_i(t)$ are known time- varying matrices with suitable dimensions. The subscript i denotes the ith sensor and L is the number of sensors.

For the upcoming results, we make the following assumptions.

Assumption 1. $w(t)$ and $v_i(t)$ satisfy the following statistical properties

$$\mathrm{E}[w(t)] = 0, \ \mathrm{E}[v_i(t)] = 0 \tag{3}$$

$$\begin{aligned}
&\mathrm{E}[w(t)w^{\mathrm{T}}(k)] = Q(t)\delta_{t,k} + \textstyle\sum_{\tau=1}^{N} Q(t,k)(\delta_{t,k-\tau} + \delta_{t,k+\tau})\\
&\mathrm{E}[v_i(t)v_j^{\mathrm{T}}(k)] = R_{ij}(t)\delta_{t,k} + \textstyle\sum_{\tau=1}^{N} R_{ij}(t,k)(\delta_{t,k-\tau} + \delta_{t,k+\tau})\\
&\mathrm{E}[w(t)v_i^{\mathrm{T}}(k)] = S_i(t)\delta_{t,k} + \textstyle\sum_{\tau=1}^{N} S_i(t,k)(\delta_{t,k-\tau} + \delta_{t,k+\tau})
\end{aligned} \tag{4}$$

where N is a known positive integer.

Remark 1. From (4), it is observed that the process noise and observation noises are N-step auto- and cross-correlated. Without loss of generality, we assume that they have the common same correlation steps for simple expression of later derivations. In practice, the process noise $w(t)$ and the observation noises $v_i(t)$ maybe have different correlated step numbers. More generally, the process noise $w(t)$ is N_0-step auto-correlated and $\overline{N_i}$-step cross-correlated with the observation noise $v_i(t)$, the observation noise $v_i(t)$ is N_i-step auto-correlated and N_{ij}-step cross-correlated with $v_j(t)$, $i \neq j$. Then, we can set $N = \max\{N_0, \overline{N_i}, N_i, N_{ij}, i, j = 1, 2, ..., L\}$ in (4) where some correlation matrices are zeros.

Assumption 2. The initial state $x(0)$ with $\mathrm{E}[x(0)] = \mu_0$ and $\mathrm{E}[(x(0) - \mu_0)(x(0) - \mu_0)^{\mathrm{T}}] = P_0$ is uncorrelated with $w(t)$ and $v_i(t)$, $i = 1, 2, \cdots, L$.

Our aim is to find the distributed fusion filter $\hat{x}_o(t|t)$ by matrix weighting sum of local filters $\hat{x}_i(t|t)$ from individual sensors based on the observations $(y_i(t), y_i(t-1), \cdots, y_i(0))$, $i = 1, 2, \cdots, L$.

3. Optimal local filter

Recently, optimal linear estimators have been designed for systems with finite-step correlated noises and packet dropout compensation based on an innovation analysis approach in [23]. So, optimal local filter can be obtained based on the results in [23] where $\xi(t) = 1$ in observation equation which means that there are no packet losses during data transmission. The following Lemmas are straight forward from [23].

Lemma 1 (*[23]*). *For systems (1) and (2) under Assumptions 1 and 2, local optimal filter $\hat{x}_i(t|t)$ and one-step predictor $\hat{x}_i(t+1|t)$ of the ith sensor subsystem are calculated by*

$$\hat{x}_i(t|t) = \hat{x}_i(t|t-1) + K_{x_i}(t|t)\varepsilon_i(t) \tag{5}$$

$$\hat{x}_i(t+1|t) = \Phi(t)\hat{x}_i(t|t) + \Gamma(t)\hat{w}_i(t|t) \tag{6}$$

where the gain matrix $K_{x_i}(t|t)$ of the state filter is given by

$$K_{x_i}(t|t) = \left[P_{\tilde{x}_i}(t|t-1)H_i^{\mathrm{T}}(t) + P_{\tilde{v}_i\tilde{x}_i}^{\mathrm{T}}(t|t-1)\right]Q_{\varepsilon_i}^{-1}(t) \tag{7}$$

The innovation sequence $\varepsilon_i(t)$ and its covariance matrix $Q_{\varepsilon_i}(t)$ are calculated by

$$\varepsilon_i(t) = y_i(t) - H_i(t)\hat{x}_i(t|t-1) - \hat{v}_i(t|t-1) \tag{8}$$

$$\begin{aligned}
Q_{\varepsilon_i}(t) = {} & H_i(t)P_{\tilde{x}_i}(t|t-1)H_i^{\mathrm{T}}(t) + P_{\tilde{v}_i}(t|t-1) + H_i(t)P_{\tilde{v}_i\tilde{x}_i}^{\mathrm{T}}(t|t-1)\\
& + P_{\tilde{v}_i\tilde{x}_i}(t|t-1)H_i^{\mathrm{T}}(t)
\end{aligned} \tag{9}$$

The estimation error covariance matrices $P_{\tilde{x}_i}(t|t)$ of the filter and $P_{\tilde{x}_i}(t+1|t)$ of the predictor are given by

$$P_{\tilde{x}_i}(t|t) = P_{\tilde{x}_i}(t|t-1) - K_{x_i}(t|t)Q_{\varepsilon_i}(t)K_{x_i}^{\mathrm{T}}(t|t) \tag{10}$$

$$\begin{aligned}
P_{\tilde{x}_i}(t+1|t) = {} & \Phi(t)P_{\tilde{x}_i}(t|t)\Phi^{\mathrm{T}}(t) + \Gamma(t)P_{\tilde{w}_i}(t|t)\Gamma^{\mathrm{T}}(t) + \Phi(t)P_{\tilde{x}_i\tilde{w}_i}(t|t)\Gamma^{\mathrm{T}}(t)\\
& + \Gamma(t)P_{\tilde{x}_i\tilde{w}_i}^{\mathrm{T}}(t|t)\Phi^{\mathrm{T}}(t)
\end{aligned} \tag{11}$$

The initial values are $\hat{x}_i(0|-1) = \mu_0$ and $P_{\tilde{x}_i}(0|-1) = P_0$. The noise filter $\hat{w}_i(t|t)$ and predictor $\hat{v}_i(t|t-1)$ with covariance matrices $P_{\tilde{w}_i}(t|t)$ and $P_{\tilde{v}_i}(t|t-1)$ are calculated by the following Lemma 2. The cross-covariance matrices $P_{\tilde{x}_i\tilde{w}_i}(t|t)$ and $P_{\tilde{v}_i\tilde{x}_i}(t|t-1)$ are calculated by the following Lemma 3.

Lemma 2 (*[23]*). *For systems (1) and (2) under Assumptions 1 and 2, local optimal filters $\hat{w}_i(t|t)$ and gain matrices $K_{w_i}(t|t-\tau)$ of the process noise $w(t)$*