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A modified multi-grid algorithm for a novel variational model to remove multiplicative noise ☆

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ABSTRACT

This paper proposes a novel variational model and a fast algorithm for its numerical approximation to remove multiplicative noise from digital images. By applying a maximum a posteriori (MAP), we obtained a strictly convex objective functional whose minimization leads to non-linear partial differential equations. As a result, developing a fast numerical scheme is difficult because of the high nonlinearity and stiffness of the associated Euler-Lagrange equation and standard unilevel iterative methods are not appropriate. To this end, we develop an efficient non-linear multi-grid algorithm with an improved smoother. We also discuss a local Fourier analysis of the associated smoothers which leads to a new and more effective smoother. Experimental results using both synthetic and realistic images, illustrate advantages of our proposed model in visual improvement as well as an increase in the peak signal-to-noise ratio over comparing to related recent corresponding PDE methods. We compare numerical results of new multigrid algorithm via modified smoother with traditional time marching schemes and with multigrid method via (local and global) fixed point smoother as well.

1. Introduction

Image restoration problem has been widely studied in the areas of image processing and computer vision. In many real world applications, a real recorded image may be distorted by many expected or un-expected random factors, which causes an unavoidable random noise. Such noise can be caused by a wide range of sources. For example, due to detector sensitivity, environmental variations, nature of radiation and transmission or quantization errors, etc. To improve the contrast, resolution, and quality of the images, image restoration is thus an important tool in modern imaging sciences and has been widely utilized in many fields such as astronomical imaging, astrophysics, biology, physics, chemistry, arts, geophysics, hydrology, remote sensing and other areas involving imaging techniques [27,32].

In many image formation models, noise is often modeling as an additive noise ϑ_1. The restoring model is then to recover u from the data

$$f = u + \vartheta_1 \tag{1}$$

There are many effective methods to handle this problem. Among the famous ones are wavelets approaches, stochastic approaches, principal component analysis based approaches and variational approaches [26].It becomes evident that variational approaches to the image de-noising problem have attracted much attention by directly approximating the reflectance of the underlying scene and yield often excellent results. There are two terms in the variational method; a regularizer and a data fidelity term. Due to the edge-preserving and noise removing properties, total variation approach has been widely utilized as a penalty function (regularizer) in the noise removal task. For more details see [2,3,12,14–16,25,37].

In practice, there are other types of noise as well such as multiplicative noise. It can also degrade an image. The topic of multiplicative noise reduction has attracted a lot of research attention since early 1980s [16]. At present, it has been extensively studied. Roughly speaking, the main multiplicative noise (speckle noise) removal techniques fall into four categories: filtering based methods in (a) spatial domain; or (b) a transform domain, for example, wavelet domain; (c) non-local filtering and (d) variational methods [16]. In this paper, we will focus on the variational approach based multiplicative noise removal problem, especially that our studies will emphasis on total variation based methods. The image formation model assumption is that the original image u has been

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corrupted by some multiplicative noise ϑ_2. The goal is then to recover u from the data

$$f = u\vartheta_2 \tag{2}$$

where f is a given recorded image and u is the restored image.

Multiplicative noise (also known as speckle) appears in various image processing applications, for example, in synthetic aperture radar, ultrasound imaging, laser imaging or in connection with blur in electronic microscopy, single particle emission computed tomography and positron emission tomography. Multiplicative noise is a signal independent, non-Gaussian and spatially dependent i.e. variance is a function of signal amplitude. In the case of multiplicative noise, the variance of the noise is higher when an amplitude of the signal is higher. In other words, noise in bright regions has higher variations and could be interpreted as features in the original image. Thus, it is one of the most complex image noises when it comes to smoothing noise without degrading true image features. It degrades the quality of the image and prevents us from interpreting valuable information of images, such as edges, textures, and point target, and therefore multiplicative noise (speckle noise) removal is often a necessary preprocessing step for the successful use of classical image processing algorithms involving image segmentation, detection and classification. Therefore, an effective method is desirable in those cases. Objectives of any method are: (a) to effectively suppress the noise in uniform regions, (b) to reduce staircase effect, (c) to preserve and enhance edges and other similar image features, and (d) to provide a visually natural appearance. We refer the reader to the literature [10,11,17,18,23,24,26–30,33–35,38–40,42,46] and references included herein for an overview of the subject.

To the best of our knowledge, there exist several variational approaches devoted to multiplicative noise removal problem. The first total variation based approach for multiplicative noise removal is the one introduced by Rudin et al. [38] which used a constrained optimization approach with two Lagrange multipliers. By applying a maximum a posteriori (MAP) estimator, Aubert and Aujol [2] proposed a functional whose minimization leads to the restored image to be recovered. Shi and Osher [43] adopted the data term of the AA-model and replace the regularizer TV(u) by TV(log$u$) and letting w = logu, they derived the strictly convex TV model (SO-model) [43]. Similarly with SO-model, Bioucas and Figueiredo converted the multiplicative model into an additive one by taking logarithms and introduced Bayesian type variational model [8]. Steidl and Teuber [44] presented a variational model consisting of the I-divergence as data fitting term and the TV-semi-norm as a regularizer. A variational model involving curvelet coefficients for cleaning multiplicative Gamma noise was introduced in [19]. Most recent works are a joint additive and multiplicative noise removal model by Chumchob et al. [15] and a higher-order MRF-based variational model for multiplicative noise reduction presented by Chen et al. [16] and speckle reduction via higher order total variation approach introduced by Feng et al. [20] and so on.

In this paper, a new total variation based image restoration model is proposed for multiplicative noise reduction via adopting the two fidelity terms of [16] and a robust non-linear multi-grid method with an improved fixed point smoother is employed for solving the associated Euler-Lagrange equation arisen from the total variation based minimization model. Moreover, the local Fourier analysis (LFA) of the underlying smoother, illustrating that it is not effective. A close study shows that this ineffectiveness is due to a few image pixels only, where the linearized coefficients differ vastly. Therefore, we propose a different smoothing strategy at these odd pixels. The local Fourier analysis shows that the modified smoother is effective. Experimental tests from real and synthetic images showing the effectiveness of the proposed model than the AA-model [2] and HMW-model [27] and in comparing to other current image restoration models. Numerical results also demonstrate that the proposed multigrid algorithm via modified smoother gives recovered images of good quality than some other existing iterative schemes.

The outline of this paper is as follows. Review of existing total variation based models for multiplicative noise removal is discussed in Section 2. In Section 3, our new model for addressing the problem is introduced. In Section 4, numerical methods are given to solve the associated non-linear Euler-Lagrange equation arising from the minimization of the proposed energy functional. We also use this section to discuss in detail the local Fourier analysis results in order to illustrate the shortcomings of the smoother, before, we propose an improved smoother. A complexity analysis is also presented in Section 4. In order to demonstrate the very good performance of the proposed model and our new modified multi-grid algorithm, some restoration results are provided in Section 5. Peak signal to noise ratio table is also given in Section 5. Section 6 concludes the paper.

2. Review of total variation based image restoration models

In the literature, total variation based regularization has been proven to be a very valuable technique for image restoration and is applied widely in many practical applications. Although, it has to combine with a suitably chosen data fidelity term in leading to good restoration results. This section first review the current two state-of-the-art denoising models namely AA-model [2] and HMW-model [27] for multiplicative noise removal problem. As can be seen, each model includes the data fidelity term (non-linear) able to offer high quality of denoised images. Later on, with these models, we will compare our model denoising results.

2.1. AA-model for multiplicative noise removal (M1)

Since we know that total variation functional TV(u) is being used on large scale since it has been introduced by Rudin et al. [38]. TV(u) is defined as

$$\mathrm{TV}(u) = \int_\Omega |Du| = \sup_{p\in C_0^1, |||p|||_\infty} \int_\Omega u\, div\, p dx$$

this leads for $L^1(\Omega)$ functions with weak first derivatives in $L^1(\Omega)$ as

$$\mathrm{TV}(u) = \int_\Omega |\nabla u| dx.$$

Using property of TV as edge preserver, Aubert-Aujol (AA) [2] proposed a non-convex Bayesian type total variation based multiplicative noise removal model, whose energy functional can be written uniformly as follows

$$\hat{u} = \arg\min_{u\in BV(\Omega)} \left\{ E(u) = \int_\Omega |\nabla u| dxdy + \gamma_1 \int_\Omega \left(\log u + \frac{f}{u} \right) dxdy \right\} \tag{3}$$

where the first term is the regularization term which imposes some prior constraints on the original image f and determines the quality of the recovery image u. Here, $BV(\Omega)$ denotes the bounded variation space and $\gamma_1 > 0$ is the regularization parameter. The second term is the image fidelity term which measures the violation of the relation between u and the observation f and is derived from MAP.

2.2. HMW-model for multiplicative noise removal (M2)

Recently, Jin and Yang [29] applied the exponential transformation $u \to e^u$ introduced by Huang et al. [27] with data fidelity term of AA-model and proposed the following restoration model