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A comment on “Translation and scale invariants of Tchebichef moments” by Hongqing Zhu [Pattern Recognition 37 (2007) 2530–2542]

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ABSTRACT

The aim of the present comment is to point out wrong claim made by (Hongqing Zhu, 2007) on the computation of scaling invariants of Tchebichef moments. We also proposed a novel discrete orthogonal moments namely Charlier moments, and demonstrate that the scaling invariants of the Charlier moments can be computed by using the method mentioned in the article of (Hongqing Zhu, 2007). Experimental results show that our inference is correct.

1. Introduction

The discrete orthogonal moments such as Chebyshev [1], Krawtchouk [2] and Hahn Moments [3] are free of the numerical integration approximations and the transformation of the image coordinate space. This property makes them superior to the continuous orthogonal moments in terms of preserving the useful analytical property and image representation capability [4]. Unfortunately, they are not natively scaling invariant, which degrades their extension applications.

The popular methods to obtain the scaling invariants of the discrete orthogonal moments are (1) image normalization; (2) making use of translation and scaling invariants of geometric moments. However, as indicated by Chong et al. [5], the scaling invariant moments obtained via the normalization method may differ from the true moments of the image because the normalization parameters may not always be accordant to the exact transformation of the scaled image. On the other hand, the method using geometric moments is time expensive due to the long time allocated to compute the polynomial coefficients.

Zhu et al. [4] proposed an approach which directly derived the scaling invariants of Tchebichef moments based on Tchebichef polynomials. The important contribution of this work to the theory of discrete orthogonal moments is the idea of constructing the native scaling invariants directly from the discrete orthogonal moments. However, the work contains a crucial mistake in the derivation of the scaling invariants. Considering the great importance of this work in the pattern recognition community, this paper points out the crucial mistake, and corrects this error by developing a novel kind of discrete orthogonal moment namely Charlier moment, which scaling invariants can be directly derived via the approach proposed by Zhu et al. [4]. The relevant comment on the crucial mistake and the corrections developed in this paper are supported by some experimental evidences. The remainder of this paper is organized as follows. Section 2 provides a brief review of Zhu's approach. Errata and comments on the approach are then provided in Section 3. Section 4 provides the definition of Charlier moments, and how to use Zhu's approach to directly derive their scaling invariants is also discussed. Experimental results are presented in Section 5, and finally conclusions are drawn in Section 6.

2. Review of the approach proposed by Zhu et al

The discrete Tchebichef polynomial of order n is expressed as [4]

$$t_n(x) = \sum_{k=0}^{n} \frac{(n+k)!}{(n-k)!(k!)^2} \langle n-N \rangle_{n-k} \langle x \rangle_k \tag{1}$$

where $\langle a \rangle_k = a(a-1)(a-2)\cdots(a-k+1)$, $k \geq 0$ and $\langle a \rangle_0 = 1$. For simplicity, Eq. (1) can be rewritten as

$$t_n(x) = \sum_{k=0}^{n} B_{n,k} \langle x \rangle_k \tag{2}$$

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with

$$B_{n,k} = \frac{(n+k)!}{(n-k)!(k!)^2}\langle n-N\rangle_{n-k} \tag{3}$$

For an image intensity function $f(x, y)$ with size of $N \times N$, the Tchebichef moment of order $n + m$ is defined as [4]

$$T_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{t}_n(x)\tilde{t}_m(y)f(x,y) \tag{4}$$

where $\tilde{t}_n(x) = \frac{t_n(x)}{\beta(n,N)}$, and $\beta(n,N) = \sqrt{2n!\binom{n+N}{2n+1}}$, here, $\binom{p}{q} = p!/q!(p-q)!$ denotes the combination number. According to Eq. (2), we have

$$\tilde{t}_n(x) = \frac{t_n(x)}{\beta(n,N)} = \sum_{k=0}^{n}\tilde{B}_{n,n-k}\langle x\rangle_k \tag{5}$$

where $\tilde{B}_{n,n-k} = \frac{B_{n,n-k}}{\beta(n,N)}$, and $\langle x\rangle_k$ can be expanded as [4,6]

$$\langle x\rangle_k = \sum_{i=0}^{k}s(k,i)x^i \tag{6}$$

where $s(k, i)$ are the Stirling numbers of the first kind satisfying the following recurrence relations:

$$s(k,i) = s(k-1,i-1) - (k-1)s(k-1,i),\ k \ge 1, i \ge 1 \tag{7}$$

with

$$s(k,0) = s(0,i) = 0 \text{ and } s(0,0) = 1 \tag{8}$$

Assume that the original image $f(x, y)$ is scaled with factors a and b, along x- and y-directions, respectively. The scaled Tchebichef moments can be defined as follows:

$$T''_{nm} = ab\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\tilde{t}_n(ax)\tilde{t}_m(ay)f(x,y) \tag{8a}$$

Using Eqs. (5) and (6), we have

$$\tilde{t}_n(x) = \sum_{k=0}^{n}\sum_{i=0}^{k}\tilde{B}_{n,n-k}s(k,i)x^i = \sum_{i=0}^{n}\sum_{k=0}^{n-i}\tilde{B}_{n,n-k}s(n-k,i)x^i$$
$$= \sum_{i=0}^{n}C(n,i)x^i \tag{9}$$

where

$$C(n,i) = \sum_{k=0}^{n-i}\tilde{B}_{n,n-k}s(n-k,i) = \sum_{k=0}^{n-i}C_k(n,i) \tag{10}$$

with

$$C_k(n,i) = \tilde{B}_{n,n-k}s(n-k,i) \tag{11}$$

and then

$$\tilde{t}_n(ax) = \sum_{i=0}^{n}C(n,i)a^ix^i \tag{12}$$

It can be easily deduced from Eqs. (9) and (12) that

$$\sum_{k=0}^{n}\lambda_{n,k}\tilde{t}_k(ax) = a^n\sum_{k=0}^{n}\lambda_{n,k}\tilde{t}_k(x) \tag{13}$$

where $\lambda_{n,n} = 1$, $\lambda_{n,k} = \sum_{r=0}^{n-k-1}\frac{-C_{n-r,k}\lambda_{n,n-r}}{C_{k,k}}$, $0 \le k < n$. Similarly,

$$\sum_{l=0}^{m}\lambda_{m,l}\tilde{t}_l(by) = b^n\sum_{l=0}^{m}\lambda_{m,l}\tilde{t}_l(y) \tag{14}$$

The relationship between the original and scaled Tchebichef moments can then be established as

$$\varphi_{nm} = \sum_{k=0}^{n}\sum_{l=0}^{m}\lambda_{n,k}\lambda_{m,l}T''_{k,l} = a^{n+1}b^{m+1}\sum_{k=0}^{n}\sum_{l=0}^{m}\lambda_{n,k}\lambda_{m,l}T_{k,l} \tag{15}$$

The following scaling invariants of Tchebichef moments can be constructed by eliminating the scale factors, a and b,

$$\psi_{nm} = \frac{\varphi_{nm}\varphi_{00}^{\gamma+1}}{\varphi_{n+\gamma,0}\varphi_{0,m+\gamma}^{\gamma+1}},\ n,m = 0,1,2,\cdots,\text{ and } \gamma = 1,2,3, \tag{16}$$

3. Our comments for the approach of Zhu et al

The crucial mistake in the approach of Zhu et al. is neglecting of the fact that the coefficient of $\tilde{B}_{n,n-k}$ is dependence on the length of finite data N. According to Eqs. (3) and (5), we have

$$\tilde{B}_{n,n-k} = \frac{B_{n,n-k}}{\beta(n,N)} = \frac{(2n-k)!}{k!((n-k)!)^2\beta(n,N)}\langle n-N\rangle_k \tag{16a}$$

It is obvious that $\tilde{B}_{n,n-k}$ depends on the length of finite data N, and so, the correct version of Eq. (9) should be

$$\tilde{t}_n(x) = \sum_{i=0}^{n}C(n,i,N)x^i \tag{17}$$

where

$$C(n,i,N) = \sum_{k=0}^{n-i}\tilde{B}_{n,n-k}s(n-k,i)$$
$$= \sum_{k=0}^{n-i}\frac{(2n-k)!}{k!((n-k)!)^2\beta(n,N)}\langle n-N\rangle_k s(n-k,i) \tag{18}$$

Since $\tilde{t}_n(ax)$ can be comprehended as a down sampling sequence with a interval from the discrete Tchebichef polynomial $t_n(x)$, $0 \le x < aN - 1$, the correct version of Eq. (12) should be

$$\tilde{t}_n(ax) = \sum_{i=0}^{n}C(n,i,aN)a^ix^i \tag{19}$$

where

$$C(n,i,aN) = \sum_{k=0}^{n-i}\frac{(2n-k)!}{k!((n-k)!)^2\beta(n,aN)}\langle n-aN\rangle_k s(n-k,i) \tag{20}$$

Consequently, the coefficient $\lambda_{n,k}$ in Eq. (13) should be

$$\lambda_{n,k} = \sum_{r=0}^{n-k-1}\frac{-C(n-r,k,aN)\lambda_{n,n-r}}{C(k,k,aN)} \tag{21}$$

It is obvious that $\lambda_{n,k}$ depends on the scale factor a, and so the invariants ψ_{nm} constructed via Eq. (16) are dependence on the factors a and b due to that the scale factors cannot be eliminated completely by Eq. (16).

4. The corrections for the approach of Zhu et al

In Section 3, we explain the reason why the aforementioned approach is not applicable to calculating the scaling invariants of Tchebichef moments. In this Section, we will correct this error by developing a novel discrete orthogonal moment namely Charlier moment, and demonstrate that the scaling invariants of Charlier moments can be directly derived via the approach proposed by Zhu et al.

Charlier polynomials with one variable $c_n^{a_1}(x)$ satisfy the following first-order partial differential equation of the form [7]

$$x\Delta\nabla c_n^{a_1}(x) + (a_1 - x)\Delta c_n^{a_1}(x) + nc_n^{a_1}(x) = 0 \tag{22}$$