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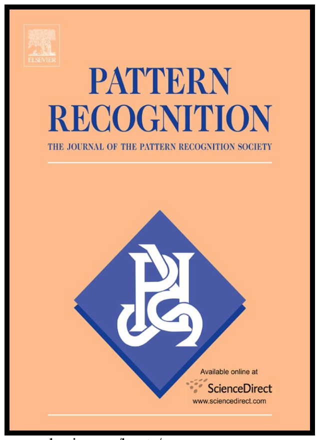

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S. Garrido-Jurado, R. Muñoz-Salinas, F.J Madrid-Cuevas, R. Medina-Carnicer

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Generation of fiducial marker dictionaries using mixed integer linear programming

S. Garrido-Jurado[1], R. Muñoz-Salinas*, F.J Madrid-Cuevas, R. Medina-Carnicer

Computing and Numerical Analysis Department, Córdoba University, Spain

Abstract

Square-based fiducial markers are one of the most popular approaches for camera pose estimation due to its fast detection and robustness. In order to maximize their error correction capabilities, it is required to use an inner binary codification with a large inter-marker distance. This paper proposes two Mixed Integer Linear Programming (MILP) approaches to generate configurable square-based fiducial marker dictionaries maximizing their inter-marker distance. The first approach guarantees the optimal solution, however, it can only be applied to relatively small dictionaries and number of bits since the computing times are too long for many situations. The second approach is an alternative formulation to obtain suboptimal dictionaries within restricted time, achieving results that still surpass significantly the current state of the art methods.

Keywords: fiducial markers, MILP, mixed integer linear programming, augmented reality, computer vision.

1. Introduction

Camera pose estimation is a common problem in numerous computer vision applications such as robot navigation [1, 2] or augmented reality [3, 4, 5], which is usually based on obtaining correspondences between environment and image points. While the use of natural features, such as key points or textures [6, 7, 8, 9], is a very popular strategy which does not require altering the environment, the use of fiducial markers is still of great importance since it provides point correspondences more robustly, efficiently and precisely.

In particular, square-based fiducial markers are the most popular in the field of augmented reality [4, 10, 11] since a single marker provides the four points required to estimate the camera pose (given that it is properly calibrated). In general, squared-based markers use an inner binary code for identification, error detection and correction.

The detection process of this type of markers can be split in two main steps. The first step is the candidate search, which consists in finding square shapes in the image that look like markers. The second step is the identification stage, where the inner codification of the candidates is analyzed in order to determine whether they really are markers, and if they belong to the considered set of valid ones, also known as dictionary.

A key aspect of such dictionaries is the inter-marker distance [10], which is the minimum Hamming distance between the binary codes of the markers, considering the four possible rotations. This distance defines the maximum number of bits that can be corrected without producing an inter-marker confusion error, i.e. a marker being erroneously identified as a different one. As a consequence, the inter-marker distance is directly related to the error correction capabilities of a dictionary. The larger the inter-marker, the lower the false negative and inter-marker confusion rates, and therefore, the higher the robustness of the process.

For instance, Figure 1 shows an example of inter-marker confusion error and the importance of large inter-marker distances. The two first markers have a short distance of only 1 *bit while the third marker has larger distances of at least* 5 *bits to the rest of markers. As it can be seen in Figures 1d,e, a single erroneous bit is enough to cause a wrong identification of the second marker. On the other hand, the third marker is correctly identified despite having a higher number of errors.*

Most related works propose their own predefined dictionary of markers with a fixed number of markers and bits, *and a constant inter-marker distance.* However, using a predefined dictionary for every application is not the optimal approach. Instead, if the number of required markers and their size is known, it is preferable to create a custom dictionary that maximizes *the inter-marker distance and, consequently,* the error detection and correction capabilities. *Although this is the tendency of the latest proposals [12, 13], they rely on heuristic approaches, none of them being optimal.*

This paper presents two novel dictionary generation methods based on the Mixed Integer Linear Programming (MILP) paradigm. The first MILP model proposed guarantees the optimal inter-marker distance for a specific number

*Corresponding author

Email addresses: i52gajus@uco.es (S. Garrido-Jurado), rmsalinas@uco.es (R. Muñoz-Salinas), fjmadrid@uco.es (F.J Madrid-Cuevas), rmedina@uco.es (R. Medina-Carnicer)

[1]Computing and Numerical Analysis Department, Edificio Einstein. Campus de Rabanales, Córdoba University, 14071, Córdoba, Spain, Tlfn:(+34)957212255