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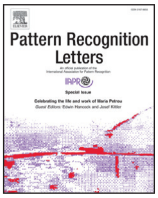

Axis estimation of thin-walled axially symmetric solids

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ABSTRACT

In this paper, a new method for axis detection of discrete thin-walled axially symmetric surfaces is presented. This method is based on the property of thin-walled axially symmetric surfaces that the minimum path of a point on the external wall to the internal wall is on a straight line passing through the axis. This working principle, since it does not require the evaluation of differential geometrical properties, makes the method robust to noise.

The proposed method has been applied in a very critical application area: axially symmetric archaeological pottery fragments, for which the evaluation of the axis is complex because of manufacturing error and of modification of the surface properties due to the action of time and weather.

The trueness of the proposed method is compared with those of the five methods presented in the literature in the analysis of real sherds of various dimensions and conditions. The proposed method demonstrates greater robustness than these methods and is shown to be promising to improve the number of sherds that can be successfully analyzed.

1. Introduction

Ceramics are the most common finds in archaeology and they can contain important information concerning the history, economy and art of the site. Because ceramic is a brittle material, ancient pottery is found reduced to fragments. In order to obtain useful information from this kind of find, they must be properly analyzed, reconstructed and classified. Pottery fragments are found in large quantities and, at the same time, they are very difficult to deal with; these are two factors that make it very expensive to investigate pottery. The traditional method used by archaeologists to study pottery consists of manual measurements and drawings and the classification is driven by a visual comparison with published data. This method is time-consuming (from ½ to 3 h for each fragment) and the results very often depend on operator subjectivity, specialization, personal skills and professional experience. As direct consequences, it is common to analyze only indicative fragments of an excavation and the results are very often not reproducible or repeatable [4].

In order to solve these problems and also due to the diffusion of 3D scanning methods in the cultural heritage, in the last two decades, efforts have been underway to develop automatic computer-based methods of sherds analysis. Most pottery is characterized by an axially symmetric geometry. This geometric property of ceramic fragments suggests evaluating the axis with the purpose of sherd analysis in general, and for pottery reconstruction in particular.

Axis detection is not a trivial process since the models of the object are not ideal axially symmetric manifolds, and real sherds are affected by manufacturing errors and are worn by the action of environmental agents acting over time. Very often sherds have small dimensions and this is a further element that produces complexity. Evaluation of the axis is a critical phase of each known method. An inaccurate estimation of the axis affects the final classification results and the archiving of the data concerning the finds. Pottery fragments are typically thin-walled axially symmetric objects whose geometric properties derive from the manufacturing process.

In this paper, a new method for detection of the axes of thin walled axially symmetric objects is proposed. The working principle is based on the property that the minimum wall thickness direction always intersects the axis of revolution. The trueness of the proposed method has been compared with that of the most important methods presented in the literature for the analysis of real fragments. It is demonstrated that the proposed method is very robust than any other method presented in the literature.

2. Related works

The automatic methods for the axis estimation are based on the analysis of some surface properties.

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The *symmetry line-based* method [3] is founded on the property of axially symmetric surfaces, whereby the symmetry line of any section curve resulting from a plane always intersects the axis of the surface. The method is robust and accurate but it can be used only for complete axially symmetric surfaces.

Normal-based methods are founded on the property of generic axially symmetric surfaces for which the normal vectors at the points of the surface intersect the axis. Pottmann et al. [12] take advantage of this property to estimate the axis in discretized axially symmetric surfaces by processing the normal at each node of it. The axis is evaluated as the straight line that minimizes its distance from the normal vectors.

Cao and Mumford [2] proposed a method based on the property according to which the center of one of the two circles, locally approximating the surface along the principal directions, and evaluated at each node of the tessellated surface, lies on the axis of the axially symmetric surface.

In the method proposed by Lao et al. [10], the axis is evaluated as one of the principal axes of inertia of the unit-mass point set of the Gaussian Map of the tessellated surface. To generate the Gaussian Map, the normal at the facets is used, whose length is proportional to the facet area. Due to its principle working, this method is suitable only for processing complete spanned cylindrical surfaces.

Since all these methods to estimate the axis of symmetry work by analyzing geometrical-differential properties, they are very sensitive to all those factors that make an object imperfectly axially symmetric.

The method proposed by Han and Hahn [6] is based on the property of rotational surfaces, for which each section perpendicular to its axis is a circle. The search for the section that is perpendicular to the axis is performed among a set of planes, choosing the plane in which the variation of the radii of curvature and of principal curvatures evaluated in the sectioning profile is minimum. The axis is defined perpendicular to this plane and anchored to the center of the circle approximating the sectioning profile. Since there is the risk of the methods for axis estimation falling within a solution that is a local minimum, a quite valid first attempt solution can be useful to initialize the search process. For this reason, Halir [5] proposed a *hybrid method* consisting of several steps. A first attempt estimation of the axes is performed by a *normal-based method*, similar to those proposed by Pottmann et al. [12] and refined by the *M-estimator method*. The final axis is obtained by using a classical *circle and line fitting* iterative refinement approach, applied starting from the first attempt estimation of the axis. Willis and Cooper [14] proposed a method that, starting from an initial estimation obtained by using the Pottmann method [12], the final axis is obtained by minimizing a specific *Energy function* based on the axis parameters and on coefficients of an assigned referring profile curve. With the aim to increase the robustness about the noise, Mara and Sablatnig [11] and Son et al. [13] proposed circle and line fitting methods without the evaluation of the differential geometrical properties. These methods differ each other for the strategy used to search the spatial directions of the axes to be analysed. Karasik and Smilansky [7] proposed a semi-automatic method where the preliminary estimation of the axis is facilitated by the operator, who selects, manually, three points that are on a quite circular section on a plane perpendicular to the axis. For this purpose, some evidence on the object, such as the rim of the sherd, is necessary for operator intervention. The estimation of the axis is obtained by an iterative method, which minimizes the projection of the centers of circular arcs resulting from sections with planes perpendicular to the first attempt solution. The final estimation is obtained by minimizing the mean width of the projected profiles on the plane containing the axis. Although the circle and line fitting methods perform high capability in axis identification in metrology applications (for example mechanical components), they are not accurate for object which are incomplete circular span, especially when they are affected by large non idealities. This is the case with archeological pottery fragments that have been affected by the action of many factors acting over time to produce surface wear, which adds to the initial manufacturing error. In addition, the limited extension of typical pottery sherds is a factor that makes it difficult to estimate the axis.

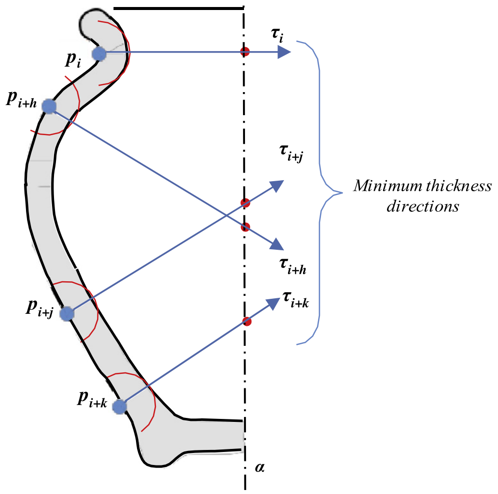

Fig. 1. Principle of methodology.

For the about mentioned reasons, the methods presented in the literature are quite unsuitable for practical applications. This will also be evidenced in the experimentation discussed in the Section 4 of the paper.

3. The proposed method

The method proposed here for axis evaluation is based on the property of ideal axially symmetric thin-walled solids that the minimum path of a point on the external wall to the internal wall is on a straight line passing through the axis $\boldsymbol{\alpha}$. So, the straight lines that pass to the axis can be found by searching for the minimum thickness line. The generic minimum thickness line can be represented by the versor ($\boldsymbol{\tau}_i$), which is defined by six coordinates $\lfloor l_{1,\tau_i}, l_{2,\tau_i}, l_{3,\tau_i}, \delta_{1,\tau_i}, \delta_{2,\tau_i}, \delta_{3,\tau_i} \rfloor$ (Plucker coordinates). The first three terms represent the directional versor ($\boldsymbol{l}_{\tau_i}$) of the line, and the last three identify the moment vector of the line with respect to the origin: $\boldsymbol{\delta}_{\tau_i} = \mathbf{p_i} \wedge \boldsymbol{l}_{\tau_i}$ ($\mathbf{p}_i$ being the point on the external wall, as in Fig. 1).

Due to the previously mentioned property, the axis $\boldsymbol{\alpha}$ (defined in terms of $\mathbf{l}_\alpha$ and $\mathbf{l}_\delta$) of a perfectly axially symmetric surface can be identified by the *thickness versor* which passes through it. For these reasons, the axis $\boldsymbol{\alpha}$ verifies the following set of m linear equations, which imposes the incidence of the m *thickness versor* with the unknown axis:

$$\boldsymbol{\delta}_\alpha \cdot \boldsymbol{l}_{\tau_i} + \boldsymbol{\delta}_{\tau_i} \cdot \boldsymbol{l}_\alpha = 0 \quad \text{with } i = 1, \ldots, m \qquad (1)$$

Since $\boldsymbol{l}_{\tau_i}$ and $\boldsymbol{l}_\alpha$ are both versors, Eq. (1) is the measures of the distances between $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}_i$ [3]. Since of the six components of $\boldsymbol{\alpha}$ only four are independent of each other, four *thickness versors*, whose coefficients are linearly independent, are required. For an imperfectly axially symmetric surface, Eq. (1) is not rigorously verified and it can be rewritten by introducing the residual error $O_i[\boldsymbol{\alpha}, \boldsymbol{\tau}_i]$:

$$\boldsymbol{\delta}_\alpha \cdot \boldsymbol{l}_{\tau_i} + \boldsymbol{\delta}_{\tau_i} \cdot \boldsymbol{l}_\alpha = O_i[\boldsymbol{\alpha}, \boldsymbol{\tau}_i] \text{ with } i = 1, \ldots, m \qquad (2)$$