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Characterization and generation of straight line segments on triangular cell grid

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ABSTRACT

In this paper we are considering straight lines and straight line segments defined by two triangle centroids in the triangular cell grid. A generation algorithm determining all the triangles (triangular cells) that are crossed by a straight line segment is proposed. There is the particular case where straight lines or straight line segments cross a vertex of the grid. We propose an arithmetical characterization of such lines and line segments.

1. Introduction

Numerous studies have dealt with the computer representation of lines and curves in the regular square grid [9,17]. The characterizations of digital straight lines have been formulated, and many algorithms for determining whether a digital arc is straight have been defined. The most popular algorithm is the generation of digital straight line (DSL) by Bresenham [1]. A study on digital straight line segments (DSS) is proposed in [16]. The digital straightness and its relationship to other the concepts of geometry, i.e., the theory of words, number theory, etc. are presented in [6].

Classical square grids are not the only grid types that have been studied. The hexagonal and triangular grids have enjoyed some attention as well. A connection exists between the cubic, the hexagonal and the triangular grids [5,10,13,18].

In this paper we are interested in triangular cell grids. Triangular grids have a theoretical as well as practical interest because of their link to triangular meshes. This work is also meant as first step into higher dimensional problems. We are especially interested in tetrahedral grids that play an important role in finite element computations in mechanics [2,4,7]. One of the specific points about triangular cell grids compared to square cell grids is that they are composed of two type of cells (*even* and *odd* triangles) and are most easily described by a three coordinate system. Those coordinate triplets are however not independent [11]. The distances for neighborhood sequences in triangular grid is stated in [14]. the characterization of one of the geometric primitives—circle is presented in [12]. Other application related works on triangular grid can be found in [8].

A generation algorithm for DSS in a triangular grid has been proposed by Freeman [3] but the DSS follow the grid model (the line segment is drawn on the edges of the grid) whereas our algorithm follows a cell model (our line segments cross cells). The algorithm in [3] generates the optimum straight-line approximation for a plotter constrained to move a unit distance at a time in one of six equi-spaced directions. In this paper our algorithm determines the cells (triangles) that are crossed by the DSS between two cell centroids. In such a definition, there is the question of what happens when the continuous line segment crosses a vertex of the grid. In this case we chose to select the triangles whose cell interiors are crossed (that can be easily changed to include all the triangles sharing the vertex). As a theoretical contribution in the field of discrete geometry, we propose an arithmetical characterization of the straight lines and straight line segments that cross a vertex. The next step will be to work on tetrahedral cell grids with line and plane traversal problems.

The organization of the paper is as follows. The preliminaries are discussed in Section 2. Section 3 presents a characterization of straight lines and straight line segments in a triangular cell grid. The algorithm is stated in Section 4. The concluding remarks are given in Section 5.

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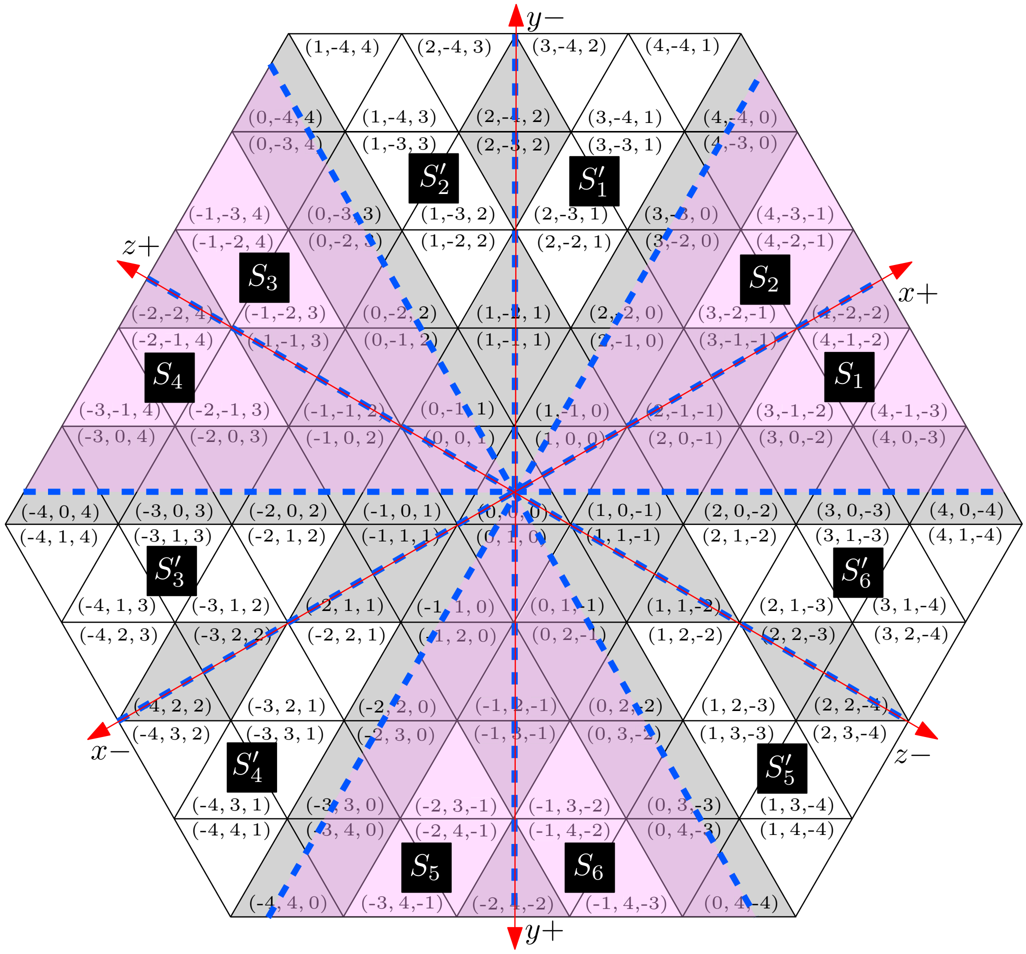

Fig. 1. The coordinate values and axes in triangular grid. The triangular grid is divided into twelve equal dodecants based on the symmetries of triangular straight line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2. Preliminaries

We are considering a triangular cell grid. Each cell is an equilateral triangle of side one. The centroids of the cells form the grid points

Each pixel of the triangular grid is a *triangle*. The centroids of the cells from the grid *points*. We suppose that one side of the cells is parallel to the Euclidean abscissa axis. A coordinate system with three coordinates has been proposed for triangular grids [15]. That way each triangle has a unique coordinate triplet. Since a triangular grid is in 2-dimensional plane, the three coordinate values are dependent on each other. There are two orientations for the cells in a triangular grid. The sum of the three coordinates is either 0, which corresponds to cells of shape △ and is termed as *even triangle*, or the sum is equal to 1, for *odd triangles* having a shape ▽. The origin cell has the coordinates (0, 0, 0). The three lines passing through the center of the origin cell, which are orthogonal to its sides, are considered as three coordinate axes *x*, *y*, and *z* (see Fig. 1). The coordinates of any cell can be determined by considering the movement which must be parallel to one of the axes. If the step is in the direction of an axis, then the respective coordinate value is increased by 1, and when the step is in the opposite direction to an axis, the respective coordinate value is decreased by 1. In this way, every triangle gets a unique coordinate triplet.

Each cell is identified by its coordinate triplet or by its centroid. There is a direct relationship between the coordinate triplets and the centroid coordinates in the regular *xy* Euclidean system. Let the coordinates of a cell be (i, j, k), then the coordinates of its centroid (c_x, c_y) in the regular *xy* Euclidean coordinate system is given by:

$$c_x = \frac{i-k}{2} \quad (1)$$

$$c_y = \frac{\sqrt{3}(i-2j+k)}{6} \quad (2)$$

We consider (continuous) straight lines and straight line segments that join cell centroids. The digital straight lines (DSL) and the digital straight line segments (DSS) in the triangular grid is the set of cells crossed by the corresponding continuous line or line segment. A straight line (or straight line segment) may intersect vertices of the triangular grid. In Fig. 2(a), the continuous straight line intersects with the vertices whereas in Fig. 2(b), it does not intersect with the vertices of grid. The corresponding intersection is termed as *vertex-cut* in this paper. In Section 3 we are proposing an arithmetical characterization of DSS and DSL with vertex-cuts.

Let the triangular grid be divided into twelve equal dodecants (i.e., the division at 30° interval w.r.t. the origin). The dodecants are marked as S_m and S'_m where $1 \le m \le 6$ (see Fig. 1). In dodecants S_m one of the coordinate triplet is negative and the other two are positive whereas in dodecants S'_m one of the coordinate triplet is positive and the other two are negative. There are similarities in the triangular straight line in dodecants S_1, S_2, S_3, S_4, S_5, and S_6 and same for dodecants $S'_1, S'_2, S'_3, S'_4, S'_5$, and S'_6. Any point in dodecant S_1 can be transformed to other similar dodecants and similarly, a point in dodecant S'_1 can be transformed to other sim-