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Asymmetric parallel 3D thinning scheme and algorithms based on isthmuses ☆

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ABSTRACT

Critical kernels constitute a general framework settled in the context of abstract complexes for the study of parallel thinning in any dimension. We take advantage of the properties of this framework, to propose a generic thinning scheme for obtaining "thin" skeletons from objects made of voxels. From this scheme, we derive algorithms that produce curve or surface skeletons, based on the notion of 1D or 2D isthmus. We compare our new curve thinning algorithm with all the published algorithms of the same kind, based on quantitative criteria. Our experiments show that our algorithm largely outperforms the other ones with respect to noise sensitivity. Furthermore, we show how to slightly modify our algorithms to include a filtering parameter that controls effectively the pruning of skeletons, based on the notion of isthmus persistence.

1. Introduction

The usefulness of skeletons in many applications of pattern recognition, computer vision, shape understanding etc. is mostly due to their property of topology preservation, and preservation of meaningful geometrical features. Here, we are interested in the skeletonization of objects that are made of voxels (unit cubes) in a regular 3D grid, *i.e.*, in a binary 3D image. In this context, topology preservation is usually obtained through the iteration of thinning steps, provided that each step does not alter the topological characteristics. In sequential thinning algorithms, each step consists of detecting and choosing a so-called simple voxel, that may be characterized locally (see [13,19,47]), and removing it. Such a process usually involves many choices, and the final result may depend, sometimes heavily, on any of these choices. This is why parallel thinning algorithms are generally preferred to sequential ones. However, removing a set of simple voxels at each thinning step, in parallel, may alter topology. The framework of critical kernels, introduced by one of the authors in [4], provides a condition under which we have the guarantee that a subset of voxels can be removed without changing topology. This condition is, to our knowledge, the most general one among the related works. Furthermore, critical kernels indeed provide a method to design new parallel thinning algorithms, in which the property of topology preservation is built-in, and in which any kind of constraint may be imposed (see [6,8]).

Among the different parallel thinning algorithms that have been proposed in the literature, we can distinguish between symmetric and asymmetric algorithms. Symmetric algorithms (see [24,32,38]) produce skeletons that are invariant under 90 degrees rotations. They consist of the iteration of thinning steps that are made of (1) the identification and selection of a set of voxels that satisfy certain conditions, independently of orientation or position in space, and (2) the removal, in parallel, of all selected voxels from the object. Symmetric algorithms, on the positive side, produce a result that is uniquely defined: no choice is needed. On the negative side, they generally produce thick skeletons, see Fig. 1.

Asymmetric skeletons, on the opposite, are preferred when thinner skeletons are required. The price to pay is a certain amount of choices to be made. Most asymmetric parallel thinning algorithms fall into three main classes:

(i) In the so-called directional algorithms (see [16,22,23,35,36,40,42–45,49,50]), each thinning step is divided into a certain number of substeps, which are each devoted to the detection and the deletion of voxels belonging to one "side" of the object: all the voxels considered during the substep have, for example, their south neighbor inside the object and their north neighbor outside the object. The order in which these directional substeps are executed is set beforehand, arbitrarily.
(ii) Subgrid (or subfield) algorithms (see [5,27,31,33,34,36,44,48]) form a second category of asymmetric parallel thinning algorithms. There, each substep is devoted to the detection and the deletion of voxels that belong to a certain subgrid, for example, all voxels that have even coordinates. Considered subgrids must form a partition of the grid. Again, the order in which

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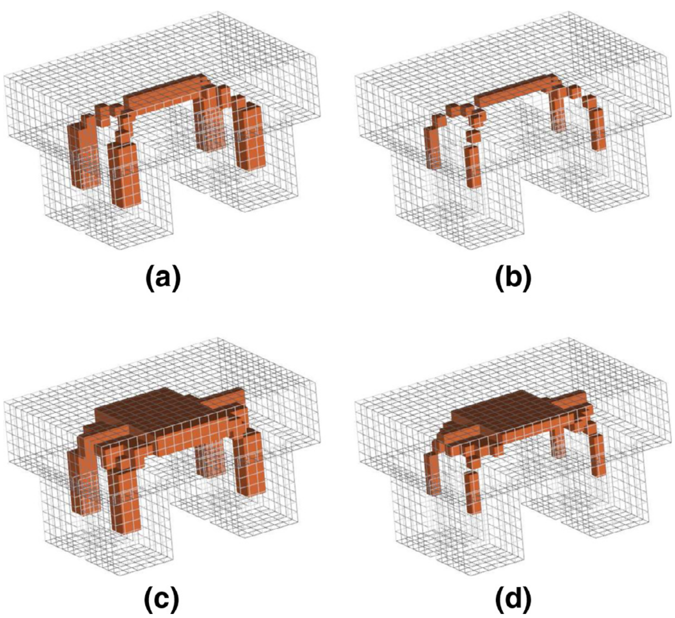

Fig. 1. Different types of skeletons. (a) Curve skeleton, symmetric. (b) Curve skeleton, asymmetric. (c) Surface skeleton, symmetric. (d) Surface skeleton, asymmetric.

subgrids are considered is arbitrary. Subgrid algorithms are not often used in practice because they produce artifacts, that is, waving skeleton branches where the original object is smooth or straight.

(iii) In a third class of algorithms, known as fully parallel algorithms (see [28,29,36,44]), the thinning step is not divided into substeps, and the same detection condition is applied to all voxels in parallel. Notice that among those, [28] and [29] do not preserve topology (see [25,26]).

Most of these algorithms are implemented through sets of masks. A set of masks is used to characterize voxels that must be kept during a given step or substep, in order to (1) preserve topology, and (2) prevent curves or surfaces to disappear. Thus, topological conditions and geometrical conditions cannot be easily distinguished, and the slightest modification of any mask involves the need to make a new proof of the topological correctness.

Our approach is radically different. Instead of considering single voxels, we consider cliques. A clique is a set of mutually adjacent voxels. Then, we identify the critical kernel of the object, according to some definitions, which is a union of cliques. The main theorem of the critical kernels framework (see [4], see also [8]) states that we can remove in parallel any subset of the object, provided that we keep at least one voxel of every clique that is part of the critical kernel, and this guarantees topology preservation. Here, as we try to obtain thin skeletons, our goal is to keep, whenever possible, exactly one voxel in every such clique. This leads us to propose a generic parallel asymmetric thinning scheme, that may be enriched by adding any sort of geometrical constraint. From our generic scheme, we easily derive, by adding such geometrical constraints, specific algorithms that produce curve or surface skeletons. To this aim, we define in this paper the notions of 1D and 2D isthmuses that permit to detect skeleton points that are important for shape reconstructibility: a 1D (resp. 2D) isthmus is a voxel whose neighborhood is “like a piece of curve” (resp. surface).

Our article is organized as follows. The first three sections contain a minimal set of basic notions about voxel complexes, simple voxels and critical kernels, respectively, which are necessary to make the article self-contained. In Section 5, we introduce our new generic asymmetric thinning scheme, and we provide some examples of ultimate

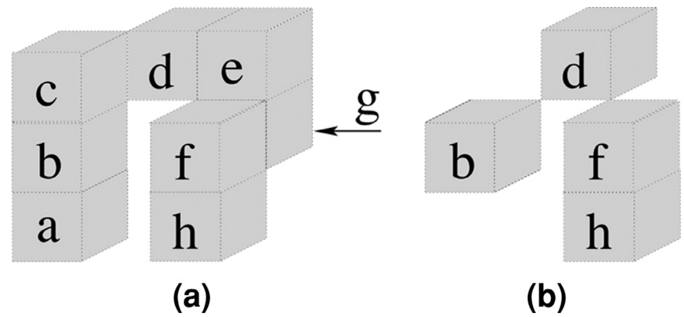

Fig. 2. (a) A complex X which is made of 8 voxels (b) A complex $Y \subseteq X$, which is a thinning of X.

skeletons obtained by using it. Section 6 is devoted to introducing and illustrating our new isthmus-based parallel algorithms for computing curve and surface skeletons. Then in Section 7, we describe the experiments that we made for comparing our curve thinning algorithm with all existing parallel curve thinning methods of the same kind. We show that our method ranks first in our quantitive evaluation. Finally, we show in Section 8 how to use the notion of isthmus persistence in order to effectively filter the spurious skeleton parts due to noise. Persistence is a criterion, easy to compute in our framework, that allows us to dynamically detect or ignore certain isthmuses.

Note: A preliminary version of this work (up to Section 6) was published in the DGCI conference proceedings [14]. Sections 7 and 8 are new.

2. Voxel complexes

In this section, we give some basic definitions for voxel complexes, see also [19,20].

Let $\mathbb{Z}$ be the set of integers. We consider the families of sets $\mathbb{F}^1_0$, $\mathbb{F}^1_1$, such that $\mathbb{F}^1_0 = \{\{a\} \mid a \in \mathbb{Z}\}$, $\mathbb{F}^1_1 = \{\{a, a+1\} \mid a \in \mathbb{Z}\}$. A subset f of $\mathbb{Z}^n$, $n \geq 2$, that is the Cartesian product of exactly d elements of $\mathbb{F}^1_1$ and $(n-d)$ elements of $\mathbb{F}^1_0$ is called a *face* or an *d-face* of $\mathbb{Z}^n$, d is the *dimension of f*. In the illustrations of this paper, a 3-face (resp. 2-face, 1-face, 0-face) is depicted by a cube (resp. square, segment, dot), see *e.g.* Fig. 4.

A 3-face of $\mathbb{Z}^3$ is also called a *voxel*. A finite set that is composed solely of voxels is called a *(voxel) complex* (see Fig. 2). We denote by $\mathbb{V}^3$ the collection of all voxel complexes.

We say that two voxels x, y are *adjacent* if $x \cap y \neq \emptyset$. We write $\mathcal{N}(x)$ for the set of all voxels that are adjacent to a voxel x, $\mathcal{N}(x)$ is the *neighborhood of x*. Note that, for each voxel x, we have $x \in \mathcal{N}(x)$. We set $\mathcal{N}^*(x) = \mathcal{N}(x) \setminus \{x\}$.

Let $d \in \{0, 1, 2\}$. We say that two voxels x, y are *d-neighbors* if $x \cap y$ is a d-face. Thus, two distinct voxels x and y are adjacent if and only if they are d-neighbors for some $d \in \{0, 1, 2\}$.

Let $X \in \mathbb{V}^3$. We say that X is *connected* if, for any $x, y \in X$, there exists a sequence $x_0, \ldots, x_k$ of voxels in X such that $x_0 = x$, $x_k = y$, and x_i is adjacent to x_{i-1}, $i = 1, \ldots, k$.

3. Simple voxels

Intuitively a voxel x of a complex X is called a simple voxel if its removal from X “does not change the topology of X”. This notion may be formalized with the help of the following recursive definition introduced in [8], see also [3,18] for other recursive approaches for simplicity.

Definition 1. Let $X \in \mathbb{V}^3$.

We say that X is *reducible* if either:

(i) X is composed of a single voxel; or

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