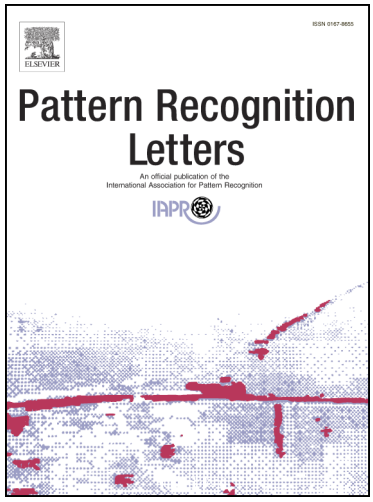

A graph-based mathematical morphology reader

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A graph-based mathematical morphology reader

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ABSTRACT

This survey paper aims at providing a "literary" anthology of mathematical morphology on graphs. It describes in the English language many ideas stemming from a large number of different papers, hence providing a unified view of an active and diverse field of research.

1. Introduction

Mathematical morphology was born almost 50 years ago (Serra, 1982), initialy an evolution of a continuous probabilistic framework (Matheron, 1975). Historically, this was the first consistent non-linear image analysis theory which from the very start included not only theoretical results, but also many practical aspects, including algorithmic ones (Soille, 1999). Despite its continuous origin, it was soon recognized that the roots of this theory were in algebraic theory, notably the framework of complete lattices (Heijmans, 1994). This allows the theory to be completely adaptable to non-continuous spaces, such as graphs. For a survey of the state of the art in mathematical morphology, we recommend (Najman and Talbot, 2010).

Graphs are generic data structures that have a long history in mathematics and have been applied in almost every scientific and engineering field, notably image analysis and computer vision (Lézoray and Grady, 2012; Grady and Polimeni, 2010). Because of their many interesting properties, a current trend is to develop the classical continuous tools from signal processing onto this kind of structures (Shuman et al., 2013).

The usefulness of graphs for mathematical morphology has long been recognized (Vincent, 1989), and the same trend as in the signal processing community can be observed here (Najman and Meyer, 2012). The objective of this paper is to offer an overview of the advantages of graphs for mathematical morphology. To reach a wider audience, we decided to express all the ideas with the least possible mathematical jargon, if possible without any equation whatsoever. We emphasize that point by using the word *reader* in the title. This paper aims at being a "literary" anthology of papers using graph in the field of mathematical morphology, describing in the English language the main ideas of many papers, pointing out where the interested researcher can find more details.

This paper is organized as follows. Section 2 describes what is a graph, what type of graphs can be encountered, and how we can build them. Section 3 explains the basis of algebraic morphology and what are the adjunctions that are used on graphs for defining elementary morphological operators. One of the most basic problem in graphs is finding paths, and section 4 gives an overview of what has been done with paths in the field. The next section 5 is divided in three parts. In the first part (section 5.1), two major morphological tools for segmentation, namely the watershed and the flat zone approach, are reviewed. The second part (section 5.2) deals with their close cousin, connective filtering. Combining these two parts togethers provide hierarchical segmentation and filtering, which is the object of section 5.3. Section 6 exhibits some links between graph-morphology and discrete calculus. Before concluding the paper, a penultimate section 7 describes several interesting structures that generalize graphs.