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Construction of Efficient and Structural Chaotic Sensing Matrix for Compressive Sensing

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Abstract

An effective sensing matrix can sample sparse or compressible signals without distortion provided that the matrix satisfies the low mutual coherence in the compressive sensing paradigm. In this work, we propose a novel structural chaotic sensing matrix (ScSM), which has the merits of both structured random sensing matrix and chaotic construction. The proposed ScSM first flips original signal, then fast and pseudo-randomly measures the flipped coefficients using a chaotic-based circulant operator, and at last, down-samples the resulting measurements to obtain the final samples. We elaborate the flipping permutation operator, chaotic-based circulant matrix, and down-sampling operator for the ScSM based on Chebyshev chaotic sequence. Moreover, the proposed ScSM is proven to yield low mutual coherence, which guarantees the desirable sampling efficiency. Experimental validations demonstrate the validity of the theory. Because of its well-designed structurally deterministic construction, the proposed ScSM has inherent superiority for storage, fast calculation, and hardware realization.

Keywords: Compressive sensing, Structural sensing matrix, Mutual coherence, Chebyshev chaotic sequence

1. Introduction

The conventional signal sampling-and-compression framework reduces redundant coefficients and maintains the crucial information of the signal to obtain fast communication and storage, where the Shannon rate is limited by the bandwidth of the original signal [1]. In the last decade, an interesting and alternative sampling theory named compressive sensing (CS) has triggered an explosion of research interest [2–5]. Unlike the typical sampling-then-compression procedure, the CS framework uses the sparse structure of most signals typically in the time or transform domain to break through the limits of Shannon's theory [5].

Mathematically, the CS implies that a high-dimensional sparse signal $\boldsymbol{x_s} \in \mathbb{R}^m$ can be accurately reconstructed from its noisy linear projection $\boldsymbol{y_s} \in \mathbb{R}^n$, where $\boldsymbol{y_s} = \boldsymbol{A} \cdot \boldsymbol{x_s} + \mathfrak{I}$, $\boldsymbol{A} \in \mathbb{R}^{m\times n}(m \ll n)$ is the sensing matrix, and $\mathfrak{I} \in \mathbb{R}^m$ represents the sensing noise. By nature, most signals can be spanned with regard to a transform domain $\{\boldsymbol{\Psi}_i\}_{i=1}^n$, i.e., $\boldsymbol{x_s} = \sum_i^n\{\boldsymbol{\Psi}_i \cdot v_i\}$, where v_i is the transform coefficient. In general, $\{v_i\}_{i=1}^n$ only has d nonzero values (or large elements) and $n-d$ zeros (or small elements). This type of signals is called a compressible signal. Let d be the sparsity of $\boldsymbol{x_s}$, i.e., $d = ||\boldsymbol{x_s}||_0 = \sum_{i=1}^n |\{x_s\}_i|^0$. Then, by substituting $\boldsymbol{x_s} = \boldsymbol{\Psi} \cdot \boldsymbol{v}$, $\boldsymbol{y_s}$ can be obtained by

$$\boldsymbol{y_s} = \boldsymbol{A} \cdot \boldsymbol{x_s} + \mathfrak{I} = \boldsymbol{A} \cdot \boldsymbol{\Psi} \cdot \boldsymbol{v} + \mathfrak{I} = \boldsymbol{\Theta} \cdot \boldsymbol{v} + \mathfrak{I}, \tag{1}$$

where $\boldsymbol{\Theta} = \boldsymbol{A} \cdot \boldsymbol{\Psi} \in \mathbb{R}^{m\times n}$, which is called the measurement matrix, and the ratio $\tau = m/n$ is the sampling rate. The process of Eq. (1) is non-adaptive, and Eq. (1) is highly ill-conditioned.

To guarantee ideal sampling without distortion, the sensing matrix $\boldsymbol{A}$ must capture and maintain the significant information of $\boldsymbol{x_s}$ during the dimensionality reduction. Beginning with the geometry, Candès *et.al* have established the famous restricted isometry property (RIP)-d on $\boldsymbol{A}$ using an isometry constant σ_d [6]. In fact, for any sparse or compressible signal, if $\boldsymbol{A}$ yields RIP-$2d$ with $\sigma_{2d} < \sqrt{2} - 1$, the exact recovery of $\boldsymbol{x_s}$ from $\boldsymbol{y_s}$ can be obtained by the l_1 convex algorithms or other algorithms such as the basis pursuit (BP) algorithm [7], BP denoising (BPDN) algorithm [8] and subspace pursuit (SP) algorithm [9].

Based on this work, researchers have further optimized the upper boundary of σ_{2d} [10, 11]. In [11], Cai *et.al* have improved it to be $\sigma_{2d} < 0.307$, which is the currently known optimal constant. The RIP plays a large role in the theoretical development of the CS framework. However, the RIP has two major shortcomings: 1) it is difficult to determine whether $\boldsymbol{A}$ yields the RIP; and 2) the RIP is only a sufficient condition to guarantee reliable recovery, which may be too loose [12]. An alternative and computable fundamental measure for the sensing matrix $\boldsymbol{A}$ is the mutual coherence.

Definition 1. The mutual coherence $u(\boldsymbol{\Theta})$ [13, 14] is the maximum absolute value of the inner product, i.e.,

$$u(\boldsymbol{\Theta}) = u(\boldsymbol{A} \cdot \boldsymbol{\Psi}) = \max_{1\le i,j\le n} \frac{|<\boldsymbol{A}_i^T, \boldsymbol{\Psi}_j>|}{||\boldsymbol{A}_i|| \cdot ||\boldsymbol{\Psi}_j||}, \tag{2}$$

where $\boldsymbol{A}_i$ and $\boldsymbol{\Psi}_j$ are the i^{th} row and j^{th} column of $\boldsymbol{A}$ and $\boldsymbol{\Psi}$, respectively.

Note that $\sqrt{\frac{n-m}{m(n-1)}} \le u(\boldsymbol{\Theta}) \le 1$, where $\sqrt{\frac{n-m}{m(n-1)}}$ is the Welch's lower bound [15]. If $m \ll n$, the lower bound $\sqrt{\frac{n-m}{m(n-1)}}$

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