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A novel equivalent circuit model for split ring resonator with an application of low phase noise reference oscillator

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ABSTRACT

This paper presents physical lumped element models for the structures employing a split-ring resonator (SRR) coupled with a pair of microstrip antennas. The results of 3D EM simulations, circuit simulations and measurement results are provided. For experimental verification, a resonator device was fabricated on an FR4 epoxy glass substrate. Mismatch between the values of resonant frequency that are predicted by the models and that are measured is less than 3%. As a benchmarking case for the proposed model, a reference oscillator was designed and implemented. A phase noise of −139.51 dBc/Hz at 3 MHz frequency offset was measured with a center frequency of 1.617 GHz.

1. Introduction

Reference oscillators are among the main building blocks in many electronic systems such as wireless communication systems [1], microprocessors [2], and sensors [3]. A frequency selective network is essential in oscillator design to determine the oscillation frequency. Resonators are usually used as the frequency selective networks. The higher the quality factor of the resonator, the better the performance of the oscillator. Recently, metamaterial-based resonators have been demonstrated as low cost and high quality resonators. Metamaterials provide the opportunity to create engineered structures, which resulted in utterly different applications, such as telecommunication [4], energy harvesting [5], sensing [6], and medical instrumentation [7]. Metamaterials interact with electromagnetic waves and can be used to control or detect them.

Split Ring Resonator (SRR) and its dual, Complementary Split Ring Resonator (CSRR), are widely used structures in metamaterial applications. They offer a very high $Q-$ factor, and exhibit high sensitivity to capacitive and inductive changes in the environment. An electrical lumped element model for these resonators is needed for effective design of circuits and systems. For this model to be used in the design process, it should also include the effects of other parts of the structure which can be transmission lines or antennas to excite resonators. An accurate lumped-element model for a CSRR loaded transmission line is provided in [8]. However, the targeted structure in this paper is an antenna-coupled SRR and the discussion will continue with its modeling hereafter. Analytical formulation for calculation of effective capacitance and inductance of the SRRs is given in the literature [9]. Lumped element models for SRRs and SRR loaded transmission lines have been demonstrated before [10–12]. Calculated capacitance and inductance values can be used in these models. Improvements to the former lumped element models were also presented [13,14]. These models are based on basic *LC* resonators with some improvements and the addition of a transmission line model. However, in addition to parameter fitting, which requires a priori information, excessive calculations are needed before using these models in a design. Furthermore, neither of them provide a direct connection between the physical dimensions of the resonator structure and the equivalent model. In addition, the whole structure including both the SRR and the effects of antennas which are used to excite the SRR have not been modeled, yet. The model proposed in this paper addresses this bottleneck, as it consists of parameters which are directly obtained from dimensions of the resonator structure and properties of the materials used in the structure and includes the effects of antennas.

This paper presents a lumped model for an antenna-coupled Split-Ring Resonator, and an oscillator utilizing the resonator. Foundation of the model and its components described in Section 2. Experimental results and their comparison with the equivalent model are provided in Section 3. Section 4 provides the verification of the model used in a reference oscillator benchmark. Finally, results and achievements are discussed in Section 5.

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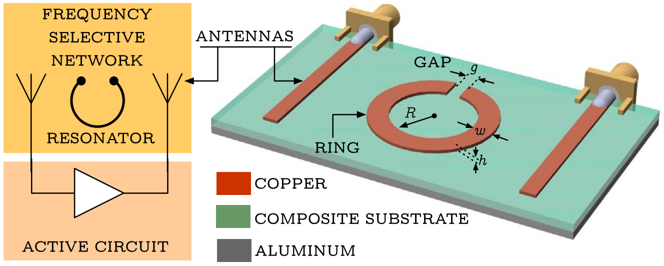

Fig. 1. Split Ring Resonator (SRR) structure with planar antennas.

2. Modeling

2.1. Electromagnetic simulations

Fig. 1 shows the structure, which comprises a metallic ring resonator and a set of microstrip antennas implemented on the same plane. Resonator has an inner ring radius of R, circular metal path having width of w, a thickness of h and a gap of g. The resonator has inductive component due to current being carried by the series path of the metallic ring and two capacitive constituents stemming from the gap and the charge distribution on the metal surface. Two different configurations of the same resonator architecture are considered: a structure with an Aluminum back plate and a structure without any backplate.

The electromagnetic behavior of the SRR structure with an inner radius of $R = 8$ mm, a width of $w = 1.25$ mm, and a gap of $g = 2.4$ mm is simulated using a commercially available electromagnetic solver (CST Microwave Studio, Darmstadt Germany). Fig. 2(a) shows the surface current density at the resonant frequency (2.1 GHz) of the SRR with no backplate, revealing a circulating current in counter-clockwise direction. Fig. 2(b) shows the reflection (S_{11}) and transmission (S_{21}) spectra, which reveal notches at the resonant frequency.

A similar resonator device having dimensions of $R = 6.85$ mm, $w = 4$ mm, $g = 1.65$ mm is designed to target a lower resonant frequency. This specific device is configured to have a 2 mm-thick aluminum plate on the backside. Fig. 2(c) shows the surface current density distribution at the resonance, exhibiting a circulating current in the counter-clockwise direction. The magnetic resonance frequency formed by the circulating current leads to a dip and a peak in reflection and transmission spectra, respectively as shown in Fig. 2(d). This current results in charge concentration across the gap. In addition to the magnetic field energy concentrated in the region enclosed by the ring, the electric field, which is created due to the charges across the gap, results in energy storage. Thus, the device shows a resonant characteristic to the perpendicular magnetic field [15]. As can be seen from the EM simulation results, device with a metallic backplate has a peak in the transmission spectra, whereas the device without backplate has a dip at resonance. The difference necessitates a specific equivalent circuit model for each configuration.

2.2. Lumped component modeling

The resonance behavior of single split-ring resonator can be modeled as a lumped LC circuit as described in [10]. The values of effective inductance and capacitance can be calculated using the following formulas as described in [9]. The total inductance is given as,

$$L_{\text{tot}} = \mu_0\left(R + \frac{w}{2}\right)\left(\log\frac{8(R + \frac{w}{2})}{h + w} - \frac{1}{2}\right) \tag{1}$$

where, μ_0 is free-space permeability. The total capacitance is;

$$C_{\text{tot}} = C_{\text{gap}} + C_{\text{surf}} \tag{2}$$

where, C_{gap} is the gap (split) capacitance, C_{surf} is the surface capacitance and these capacitances are calculated as,

$$C_{\text{gap}} = \varepsilon_0\frac{hw}{g} + C_0 \qquad C_0 = \varepsilon_0(h + w + g) \tag{3}$$

$$C_{\text{surf}} = \frac{20\varepsilon_0(h + w)}{\pi}\log\frac{4R}{g} \tag{4}$$

where, ε_0 is free-space permittivity, and C_0 is the correction to the parallel plate capacitance due to the fringe fields. The resonant frequency is calculated as;

$$f_0 = \frac{1}{2\pi\sqrt{L_{\text{tot}}C_{\text{tot}}}} \tag{5}$$

Using the design geometries, equivalent inductance and capacitance values of the SRR used in backplate configuration are calculated as $L_{\text{tot}} = 26.3$ nH and $C_{\text{tot}} = 392$ fF. The resultant resonant frequency of this device is $f_0 = 1.6$ GHz. For the SRR with no backplate equivalent parameters are calculated as $L_{\text{tot}} = 37.8$ nH and $C_{\text{tot}} = 161$ fF, respectively. This device has a calculated resonant frequency of $f_0 = 2$ GHz.

Once the effective capacitance and inductance values of the structure are calculated by using physical dimensions, these values can be directly used in the LC model to emulate the core of the resonator. However, SRRs are typically used in applications where they are embedded with other passive elements such as transmission lines and antennas. Moreover, when a filter or an oscillator application is aimed, an interface between the overall network of the resonator, including peripheral passives, and the active circuitry must be constructed. Therefore, the complete response of the network including antennas and transmission lines is needed.

2.2.1. SRR with a backplate

In this configuration, the aluminum backplate converts the microstrip lines into open circuited microstrip stubs. When the structure is examined in the absence of SRR, it can be considered as a microstrip coupled line directional coupler with open circuited through and isolated ports, and the coupled port is the second port in our structure. However, this approach is just used to model the structure. It does not suggest that the physical structures are the same as the components used in the model. Since the spacing between the lines is very large, coupling is very weak. When the SRR is added to the structure, it transfers power between lines, and a peak is observed in transmission spectrum at resonance. Off-resonance transmission will be very low.

When this operation principle is considered, the model should contain a coupled line directional coupler, an RLC resonator, and a magnetic coupling between these two circuits. The proposed lumped element model for this structure is depicted in Fig. 3(a), where R_{tot}, L_{tot}, and C_{tot} form the core of the resonator. Values of the L_{tot} and C_{tot} are calculated by using Eqs. (1) and (2), respectively. At the resonance, equivalent impedance converges to the value of R_{tot}, which controls the quality factor of the resonator. RLC resonator is connected in series with the signal path, thereby, transmission is ensured solely in vicinity of the resonance band. 50Ω microstrip coupled lines (Directional coupler-1) model the antennas having equal lengths. Isolated and through ports of Directional Coupler-2 are left open circuit to obtain the open-end effect in the antennas. Ideal transformers (TR1 and TR2) are added to the model to include the magnetic coupling between the resonator and the antennas. These transformers are placed between the directional couplers and the resonator core, to emulate the physical layout of the resonator with respect to the antennas. Coupling coefficient of the transformer is used as a fit parameter in the model, and it has no effect on the resonant frequency. It just determines the ratio of power transmitted to the resonator, thus, the magnitude of the peak in the transmission.

2.2.2. SRR without a backplate

In the absence of the back plate, microstrip lines behave as monopole antennas. Fair amount of transmission occurs between the