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# An aggregating based model order reduction method for power grids

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ABSTRACT

Simulation of power girds has become increasingly computationally expensive. In this paper, we propose a Model Order Reduction (MOR) method for power grids by extending the existing Aggregating based MOR (AMOR) method. In the proposed method, besides resistors and capacitors, current sources are also aggregated to improve MOR efficiency. Moreover, pre-partition and parallelization techniques are employed to decrease reduction time. Numerical results demonstrate that compared to original circuits, the scale of power grids is greatly reduced without much loss of accuracy. The reduced-order models are especially useful in the multiple simulations of different working modes or different environment corners.



## 1. Introduction

The continuous scale growth of power grids has made their design and analysis an extremely computationally expensive task for VLSI design. In the past decades, many techniques have been developed to improve the efficiency of power grid simulation, such as Krylov-subspace based method [1], domain decomposition method [2], and multigrid method [3,4].

Recently, however, the analysis of power grid circuits faces new challenges brought by the trends of advanced technology. The first challenge is from the introduction of on-chip Low Dropout Regulators (LDOs) [5]. This introduction makes the verification of power grids more complicated, since the linear passive components in power grids are now coupled with the nonlinear LDOs. Traditional techniques such as multigrid-like methods have difficulties in dealing with nonlinear circuits, and thus cannot reduce networks with nonlinear components as a whole. On the other hand, in the state-of-the-art low power designs, circuits often operate at multiple working modes. This more complicated dynamic situation also poses new challenge to traditional simulation methods, since simulation has to start over every time for a different mode. In such context, even though some transient simulation methods such as [6] have successfully reduced the time for power grids analysis, the overall time for multiple simulations is still significant.

Model Order Reduction (MOR) techniques can efficiently deal with the aforementioned challenges by reducing the size of linear subcircuits. Consisting of the linear reduced-order subcircuits and the rest nonlinear components, the overall reduced-order circuit models can then be reused in multi-mode or multi-corner context to accelerate simulation. Among all the MOR methods, Aggregating based MOR (AMOR) [7] is an effective method for the MOR of many-terminal interconnect circuits. It is based on the observation that adjacent nodes of interconnect circuits with almost the same voltage can be aggregated to *super nodes*. AMOR first partitions the nodes into groups using spectral partition algorithm and then aggregates the nodes with almost the same potential into super nodes by an aggregating procedure. This adaptive strategy can generate a reliable reduced-order model that is close to the original circuit in structure and in function. Unlike traditional Krylov-subspace based MOR methods which make the size of reduced-order models proportional to the inputs for moment matching, the efficiency of AMOR will never degrade as the number of terminals increases. AMOR also overcomes the weakness of elimination based MOR methods (e.g., PACT [8], SparseRC [9] and SIP [10]), which would introduce a lot of fill-ins and offset the efficiency gained by node number reduction. The experiments in [7] show that AMOR achieves higher efficiency and accuracy than PACT and SIP. Moreover, since the values of the resistors and capacitors are always positive, the reduced-order models generated by AMOR are passive and physically realizable, and, hence, more applicable in the downstream simulations.

Different from other interconnect circuits, power grids contain a large number of current sources representing loads. Unfortunately, in the partition procedure of AMOR, nodes connected with current sources are regarded as *ports*. Since ports are not aggregated during

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reduction, the reduction efficiency of AMOR for power grids is quite limited. Furthermore, AMOR gradually partitions the whole circuit into smaller blocks by recursively calling the spectral partition algorithm, which causes severe stack pressure for extremely large-scale power grids. Due to the low computational efficiency, AMOR is not sufficient to deal with large-scale power grids.

In this paper, we propose an efficient MOR method PGMOR (Power Grids' Model Order Reduction) for power grids by extending AMOR. PGMOR keeps the stability, passivity and efficacy of AMOR but becomes more suitable for power grids. Besides resistors and capacitors, current sources are also aggregated to improve the MOR efficiency. Moreover, large-scale power grids are pre-partitioned into smaller blocks by traditional min-cut partition method. The smaller blocks are then partitioned into finer groups by spectral partition algorithm in parallel. We also introduce priority queue to manage the partitioned blocks. This introduction avoids the recursive call of spectral partitioning algorithm and alleviates stack pressure for large-scale circuits. Numerical results demonstrate that compared to original circuits, the scale of power grids is greatly reduced without much loss of accuracy. Here we need to note that PGMOR is a pure MOR method which does not include the simulation part. The generated reduced-order models can be simulated by commercial simulator such as HSPICE. Moreover, they can be reused for simulations with multiple working modes, and the multigrid-like simulation methods such as [6] can use the reduced-order models to gain further speed-ups.

The rest of the paper is organized as follows. In Section 2 we give a brief review of AMOR, and then present our proposed PGMOR method in Section 3. The efficiency of our method is demonstrated by several examples in Section 4. In Section 5, we conclude the paper.

## 2. Background

The core of AMOR method includes two procedures: spectral partition and aggregation. As shown in Fig. 1, AMOR first partitions the nodes into groups using spectral partition algorithm. The nodes belonging to the same group are supposed to have similar potentials. Then these nodes are merged into super nodes by an aggregating procedure. In this section, we give a more detailed introduction to the spectral partition algorithm and aggregation procedure in AMOR.

### 2.1. Spectral partition algorithm

Spectral partition algorithm refers to the partition method based on the spectral analysis of circuit structures. It aims to partition the nodes of parasitic RC network into groups so that the voltages of adjacent nodes in the same group are almost the same.

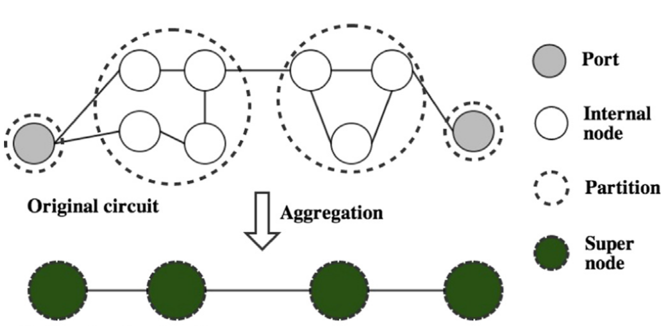


**Fig. 1.** Illustration of the AMOR method.

A parasitic RC network can be described by an undirected graph $G$ with the vertices denoting the nodes of circuits and weights of edges representing the admittance between two corresponding nodes. The effects of capacitors are neglected in the weights due to the considerations in [7].

The partition process is then based on spectral analysis of the Laplacian matrix $L$ of graph $G$. In the partition process, the eigenvector $q$ corresponding to the second smallest eigenvalue of $L$ is computed. It provides a one-dimensional placement of the circuit nodes, in which the coordinates of the nodes are the corresponding values in $q$. After obtaining the coordinates of nodes in the one-dimensional placement, the distance between two nodes can be viewed as a measure of the impedance between them. Since the nodes connected together with large admittance (or small impedance) tend to be equipotential nodes, farther distance indicates weaker correlation between them. Therefore, after sorting the eigenvector $q$, we can obtain the best partition position of the nodes by finding the biggest gap in the one-dimensional placement.

### 2.2. Aggregation procedure

After the partition result is obtained, AMOR further employs an aggregating procedure to derive the reduced-order model. In the aggregation procedure, nodes belonging to the same group are aggregated to super nodes, with resistors and capacitors connected to them being merged at the same time.

The aggregation operations on resistors and capacitors depend on their topological connections. The RCs connected between nodes of the same group are neglected. On the other hand, the RCs between ground and the nodes from the same partition are aggregated and then connected between the corresponding super nodes and ground. Finally, the RCs between the nodes of different partitions are aggregated and then connected between the super nodes that represent the corresponding partitions.

## 3. The proposed method

In this section, we demonstrate how PGMOR can effectively tackle the MOR of power grids that AMOR fails to handle. On one hand, PGMOR aggregates nodes connected to current sources, which significantly increases the percentage of merged nodes in power grids. Thus, the reduction degree of circuits is greatly improved. On the other hand, PGMOR employs a pre-partition strategy, which greatly decreases the time for eigenvector calculation and enables parallel computing. The application of priority queue to alleviate stack pressure for large-scale power grids is also included in this section.

### 3.1. Current source combination algorithm

Power grid circuits contain a large number of current sources that represent loads. In some power grid circuits, the number of current sources can reach about 1/3 of number of resistors. Unfortunately, as shown in Fig. 2, in the partition procedure of AMOR, the nodes connected with current sources are regarded as ports. Since the ports are not aggregated during reduction, the reduction efficiency of the power grids by AMOR is quite limited.

In the proposed method, we do not preserve the nodes connected to current sources as ports any more, as shown in Fig. 2. These nodes can be aggregated during the aggregation procedure. Just as the RC combination in AMOR's aggregation procedure, we employ a current source combination algorithm to merge the current sources connected to nodes belonging to the same partition.

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