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Finite-time Consensus of Heterogeneous Multi-agent Systems with Linear and Nonlinear Dynamics

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Abstract In this paper, the finite-time consensus problems of heterogeneous multi-agent systems composed of both linear and nonlinear dynamics agents are investigated. Nonlinear consensus protocols are proposed for the heterogeneous multi-agent systems. Some sufficient conditions for the finite-time consensus are established in the leaderless and leader-following cases. The results are also extended to the case where the communication topology is directed and satisfies a detailed balance condition on coupling weights. At last, some simulation results are given to illustrate the effectiveness of the obtained theoretical results.

Key words Finite-time consensus, heterogeneous multi-agent systems, linear and nonlinear dynamics

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Distributed control of networked multi-agent systems is an important research field due to its important role in a number of applications, such as the formation control of multiple robotics, attitude alignment of satellite clusters, cooperative control of unmanned aerial vehicles, target tracking of sensor networks, and so on[1−5].

The consensus of multi-agent systems means that the states of all the agents converge to a common value by applying effective consensus protocols. The convergence speed is an important factor to evaluate the performance of consensus protocols of multi-agent systems. Lots of researchers found that the convergence rates can be influenced by the second smallest eigenvalue of the interaction graph Laplacian. Some results have been developed on how to enhance the convergence speed of multi-agent system in recent years. Reference [6] found that the second smallest eigenvalue of a regular network can be increased greatly via changing the inter-agent information flow without increasing the total number of the network links. Reference [7] proposed the asymptotic and per-step convergence factors as measures of the convergence speed. One can increase convergence speed with respect to the linear protocols by maximizing the algebraic connectivity of interaction graph, but the consensus can never be reached in a finite time. However, in many practical applications, consensus can be achieved in a finite time is often required, such as when high precision performance and strict convergence time are required, when the control accuracy is crucial, etc. Reference [8] proved that if the interaction topology of a multi-agent system is connected in a sufficiently large time intervals, the proposed protocols can solve the finite-time consensus problems. Reference [9] studied the finite-time consensus of a multi-agent system using a binary consensus protocol.

However, all the aforementioned multi-agent systems were homogeneous, which means that all the agents have the same dynamics. In practical systems, the dynamics of agents may be quite different because of various restrictions in complicated environments or the different task divisions. For example, in a robot football match, according to the different role tasks, the forward robots and the midfield robots are in charge of shooting, the full back robots are responsible for the defense of particular areas. Each robot in a team has a different function and different dynamics. The team is a system with mixed robots, and can be seen as a heterogeneous multi-agent system. Because of the important application in many practical areas, some researchers start to pay attention to the heterogeneous multi-agent systems. Tanner et al.[10] studied the cooperation problems of a kind of heterogeneous mobile robot system composed of a group of unmanned ground vehicle (UGV) and a group of unmanned aerial vehicle (UAV). Consensus of velocity can be achieved based on specific communication protocols. Reference [11] studied the stationary consensus of discrete-time heterogeneous multi-agent systems with communication delays. However, the consensus protocols are convergent asymptotically. It is worthy of noting that the extension of finite-time consensus algorithms from the homogeneous case to the heterogeneous case is nontrivial. Only a few researches have studied the finite-time consensus of heterogeneous multi-agent systems. For example, [12−13] studied the finite-time consensus problem of heterogeneous multi-agent systems composed of first-order and second-order integrator agents. Several classes of nonlinear consensus protocols were constructed.

However, all the aforementioned heterogeneous multi-agent systems were composed of linear dynamics agents. In reality, nonlinear phenomena widely exist in the mechanical systems, such as teleoperation systems, attitudes of spacecraft and robotic manipulators, and the control accuracy is often crucial. To our best knowledge, there is still no literature concerning the finite-time consensus of heterogeneous multi-agent systems composed of both linear and nonlinear agents.

Inspired by the aforementioned discussion, we investigate the finite-time consensus of heterogeneous multi-agent systems composed of linear first-order, second-order integrator agents and nonlinear Euler-Lagrange (EL) agents. The main contributions of this paper can be summarized as follows:

1) We study a kind of heterogeneous multi-agent systems, which is not considered by any existing literature.

2) Some nonlinear heterogeneous control protocols are proposed, such that the heterogeneous multi-agent systems can reach a consensus in a finite time.

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3) It is shown that the investigation of this heterogeneous multi-agent system presents a unified viewpoint to study the finite-time consensus problem.

4) The results are also extended to a class of detailed balance networks and leader-following systems. In addition, linear consensus protocol is mentioned as a special case of the proposed results. The simulations demonstrate that the obtained theoretical results are effective and the heterogeneous multi-agent system has a faster convergence speed under the proposed nonlinear consensus protocol than the corresponding linear protocol.

Notation. Throughout this paper, $\mathbf{R}$ and $\mathbf{R}^+$ stand for the sets of real numbers and positive real number, respectively. $\mathbf{R}^n$ denotes the n-dimensional real vector space. $\mathbf{R}^{n\times n}$ is the set of $n\times n$ matrices. Superscript T stands for the transpose of a matrix or vector. $\|\cdot\|$ denotes the Euclidean norm. $\mathrm{sig}(x)^\alpha=\mathrm{sgn}(x)\,|x|^\alpha$, where $\mathrm{sgn}(\cdot)$ is the sign function.

1 Preliminaries

1.1 Graph theory

In this subsection, we first review some basic notations of graph theory[14]. For a multi-agent system, agents and information exchange among them are modeled by an undirected weighted graph $G=\{V,\,E,\,A\}$, where $V=\{v_i\,|i\in\Gamma\}$ is the set of agents with $\Gamma=\{1,\,2,\,\cdots,\,N\}$, $E\subseteq V\times V$ is the set of edges and A is the corresponding weighted adjacency matrix. The adjacency matrix $A=[a_{ij}]\in\mathbf{R}^{n\times n}$ is defined such that $a_{ij}>0$ if $(v_j,\,v_i)\in E$, and $a_{ij}=0$ otherwise. The set of neighbors of agent v_i is denoted by $N_i=\{v_j:\ (v_j,\,v_i)\in E\}$. The degree of agent v_i is defined as $\deg(v_i)=d_i=\sum_{j=1}^n a_{ij}=\sum_{j\in N_i}a_{ij}$. Then the degree matrix of graph G is $D=\mathrm{diag}\{d_1,\,\cdots,\,d_n\}$ and the Laplacian matrix is $L=D-A$. A directed graph G is said to be detailed balanced if there exist some scalars $\omega_i>0\,(i\in\Gamma)$ such that $\omega_i a_{ij}=\omega_j a_{ji}$ for all $i,\,j\in\Gamma$. We define one agent as the leader that can only send information to other agents but cannot receive any information from them.

1.2 Problem formulation

Suppose that a heterogeneous multi-agent system consists of linear first-order, second-order integrator agents, and nonlinear EL agents. The number of agents is N. Without loss of generality, we assume that the number of first-order integrator agents is l $(l<N)$, labeled from 1 to l, the second-order agents are labeled from $l+1$ to m, and the EL agents are labeled from $m+1$ to N. The linear first-order, second-order and nonlinear EL agents are respectively described as follows:

$$\dot{x}_i(t)=u_i(t),\quad i\in\Gamma_l \tag{1}$$

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ \dot{v}_i(t)=u_i(t)\end{cases},\quad i\in\Gamma_m\backslash\Gamma_l \tag{2}$$

$$\begin{cases}\dot{x}_i(t)=v_i(t)\\ M_i(x_i)\,\dot{v}_i+C_i(x_i,\,v_i)\,v_i=u_i(t)\end{cases},i\in\Gamma_N\backslash\Gamma_m \tag{3}$$

where $x_i\in\mathbf{R}^n$, $v_i\in\mathbf{R}^n$ and $u_i\in\mathbf{R}^n$ are respectively the position, velocity and control input of agent i, and $\Gamma_l=\{1,\,2,\,\cdots,\,l\}$, $\Gamma_m=\{1,\,2,\,\cdots,\,m\}$, $\Gamma_N=\{1,\,2,\,\cdots,\,N\}$.

The objective of this paper is to design consensus protocols for the heterogeneous multi-agent system (1) ∼ (3) such that the state consensus can be achieved in a finite time.

1.3 Key lemmas and properties

In this subsection, some important lemmas and properties are given.

Lemma 1[15]**.** Suppose function $\varphi:\mathbf{R}^2\to\mathbf{R}$ satisfies $\varphi(x_i,\,x_j)=-\varphi(x_j,\,x_i)$, $i\neq j$, $i,j\in\Gamma_N$. Then for any undirected graph G and a set of numbers $y_1,\,y_2,\,\cdots,\,y_N$, one has

$$\sum_{i=1}^N\sum_{j\in N_i}a_{ij}y_i\varphi(x_j,\,x_i)=\\ -\frac{1}{2}\sum_{(v_i,\,v_j)\in E}a_{ij}(y_j-y_i)\,\varphi(x_j,\,x_i)$$

Lemma 2[16] **(LaSalle′s invariance principle).** Let Ω be a compact set such that every solution to the system $\dot{x}=f(x)$, $x(0)=x_0$ starting in Ω remains in Ω for all $t\geq t_0$. Let $V:\Omega\to\mathbf{R}$ be a time independent locally Lipschitz and regular function such that $D^+V(x(t))\leq 0$, where D^+ denotes the upper Dini derivative. Define

$$S=\left\{x\in\Omega:D^+V(x)=0\right\}$$

Then every trajectory in Ω converges to the largest invariant set, M, in the closure of S.

Next, the homogeneity with dilation is given for the finite-time convergence analysis. For details, one can refer to [17].

Consider the autonomous system

$$\dot{x}=f(x) \tag{4}$$

where $f:D\to\mathbf{R}^n$ is a continuous function with $D\subset\mathbf{R}^n$.

A vector field $f(x)=(f_1(x),\,f_2(x),\,\cdots,\,f_n(x))^{\mathrm{T}}$ is homogeneous of degree $\kappa\in\mathbf{R}$ with dilation $r=(r_1,\,r_2,\,\cdots,\,r_n)$, $r_i>0$ $(i\in\Gamma_n=\{1,\,\cdots,\,n\})$, if, for any $\varepsilon>0$,

$$f_i(\varepsilon^{r_1}x_1,\,\varepsilon^{r_2}x_2,\,\cdots,\,\varepsilon^{r_n}x_n)=\varepsilon^{\kappa+r_i}f_i(x),\quad i\in\Gamma_n$$

Lemma 3[18]**.** Suppose that system (4) is homogeneous of degree $\kappa\in\mathbf{R}$ with the dilation $(r_1,\,r_2,\,\cdots,\,r_n)$, the function $f(x)$ is continuous and $x=0$ is asymptotically stable. If the homogeneity degree $\kappa<0$, then system (4) is finite-time stable.

Remark 1. Similar to the analysis of [19], one can easily see that the heterogeneous multi-agent system (1) ∼ (3) can achieve consensus in a finite time, if the system (1) ∼ (3) with $(x_1,\,\cdots,\,x_N,\,v_{l+1},\,\cdots,\,v_N)$ is homogeneous of degree $\kappa<0$ with dilation $(\underbrace{R_1,\,\cdots,\,R_1}_{N},\,\underbrace{R_2,\,\cdots,\,R_2}_{N-l})$ and can achieve consensus asymptotically.

We revisit the well-known properties of EL systems as follows. One can refer to [20] for more details.

Property 1. The inertia matrix $M(x)$ is a symmetric positive-definite function, and there exist positive constants m_1 and m_2 such that $m_1I\leq M(x)\leq m_2I$.

Property 2. The matrix $\dot{M}(x)-2C(x,\,v)$ is skew-symmetric.

If matrix A satisfies $A=-A^{\mathrm{T}}$, it is called skew symmetric. Property 2 means that for an arbitrary vector g,

$$g^{\mathrm{T}}\left(\dot{M}(x)-2C(x,\,v)\right)g=0$$

Property 3. There exist positive scalars K_c such that $\|C(x,\,v)\|\leq K_c\,\|v\|$.