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Nonlinear Control for a Model-scaled Helicopter with Constraints on Rotor Thrust and Fuselage Attitude

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Abstract A nonlinear control is proposed for trajectory tracking of a 6-DOF model-scaled helicopter with constraints on main rotor thrust and fuselage attitude. In the procedure of control design, the mathematical model of helicopter is simplified into three subsystems: altitude subsystem, longitudinal-lateral subsystem and attitude subsystem. The proposed control is developed by combining the sub-controls for the corresponding subsystems. The sub-controls for altitude subsystem and longitudinal-lateral subsystem are designed with hyperbolic tangent functions to satisfy the constraints; the sub-control for attitude subsystem is based on backstepping technique such that fuselage attitude tracks the virtual control for longitudinal-lateral subsystem. It is proved theoretically that tracking errors are ultimately bounded, and control constraints are satisfied. Performances of the proposed controller are demonstrated by simulation results.

Key words Nonlinear control, trajectory tracking, helicopter control, saturated control

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Trajectory tracking control design for a 6-DOF autonomous model-scaled helicopter has becomes an interesting and challenging task in recent years, because of the nonlinearities and couplings in its dynamic model[1−2]. Some representative researches include linear control[3], approximate feedback linearization[2], backstepping[4−5], robust H_∞ control[6−7], composite nonlinear feedback[8], and model predictive control[9].

Traditionally, nonlinear trajectory tracking controls for helicopters are mainly based on some assumptions: 1) constant rotational rate of rotors, 2) simplified expressions for rotor thrusts in case of low fuselage velocity and acceleration, and 3) negligence of small coupling terms (or small body forces). To support the assumptions, however, some other significant issues require further consideration. The desired main rotor thrust should be subjected to saturation; otherwise, excessively large main rotor thrust would result in large acceleration of the fuselage, and the simplified expressions for rotor thrusts are unreasonable. Besides, large rotor thrust requires large collective pitch, increasing drag forces exerted on the rotor blades and deteriorating the assumption of constant rotational rate. Moreover, attitude of the fuselage should be bounded securely for the reason that aggressive attitude often leads to uncontrollability in case of constraint on rotor thrust.

Many early works on saturated control[10−13] presented the fundamental principles and applications. Saturated control for nonlinear systems was then developed and summarized[14−17]. Saturated control strategies were applied to some specific projects, such as 3-DOF VTOL aircraft[18−19], linear motor system[20], and inverted pendulum[21]. However, researches on saturated control for trajectory tracking of 6-DOF helicopter were relatively rare. Controllers for helicopters subject to input constraints were often designed partially saturated[4, 22−23] due to nonlinearities and couplings in mathematical modeling.

Generally, saturated control are designed by using non-smooth saturation functions, which impedes analytical solution for derivatives of virtual controls, and application of Barbalat lemma to stability analysis. To overcome the troubles resulted from non-smooth saturation functions, it is necessary to apply some smooth saturation functions. Recently, a smooth hyperbolic saturated control has been designed for a 3-DOF VTOL aircraft[18]; stability results of the closed-loop system were simple to prove, and the control algorithm was easy to implement.

Enlightened by the simple smooth saturated control for 3-DOF aircraft[18], we propose a nonlinear control based on smooth saturation function for a fully 6-DOF model-scaled helicopter under constraints of main rotor thrust and fuselage attitude. The constraints are addressed by using bounded and continuously differentiable hyperbolic tangent functions. The full nonlinear model of the helicopter plant is divided into three subsystems, and the proposed control is designed by combining the sub-controls developed for the corresponding subsystems. The main contributions of this paper include: 1) The result of a 3-DOF (lateral, altitude and roll) VTOL aircraft[18] is extended to a 6-DOF helicopter; 2) Exponential stability and local input-to-state stability (LISS)[24] of the smooth saturated control system are discussed in detail; 3) Neglected coupling terms are considered theoretically in the stability analysis of the closed-loop system; 4) Time derivatives of virtual controls are presented in explicit forms without using differentiators to reduce the amount of calculation significantly.

This paper is organized as following: some useful preliminaries are reviewed in Section 1; the problem of trajectory tracking for helicopter subject to constraint on main rotor thrust and fuselage attitude is formulated in Section 2; detailed control design procedure is proposed in Section 3; the stability concerning the closed-loop system with neglected terms is analyzed in Section 4; simulation results are presented in Section 5 to illustrate the performances of the designed controller; this paper is concluded in Section 6.

1 Theoretical preliminaries

Throughout this paper, $|\cdot|$ is defined as the absolute value for real numbers; $\|\cdot\|$ is defined as the Euclidean norm for vectors, and the induced Euclidean norm for matrices.

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The conventional non-smooth saturation function is given by

$$\mathrm{sat}_\epsilon(x)=\begin{cases} x, & \text{if } |x|<\epsilon \\ \epsilon, & \text{if } |x|\geq\epsilon \end{cases}$$

which often brings the difficulties of 1) solving analytically the derivatives of the virtual control, and 2) applying Barbalat lemma to the stability analysis. To overcome such troubles, the hyperbolic tangent function

$$\tanh(s):=\frac{\mathrm{e}^s-\mathrm{e}^{-s}}{\mathrm{e}^s+\mathrm{e}^{-s}} \tag{1}$$

is utilized as the saturation function in this paper. As the saturation function, the hyperbolic tangent function is continuously differentiable. Some properties of the hyperbolic function are listed in Appendix.

The main theoretical results of this paper are based on the following proposition[18].

Proposition 1. The non-trivial solution of system

$$\begin{cases} \dot\gamma_1 &= \gamma_2 \\ \dot\gamma_2 &= -\alpha\tanh(k\gamma_1+l\gamma_2)-\beta\tanh(l\gamma_2) \end{cases} \tag{2}$$

is globally asymptotically stable for any positive numbers α, β, k and l.

In this paper, Proposition 1 is extended into a vector form. Before the extension, it is necessary to define the vector hyperbolic function.

Definition 1. The hyperbolic tangent function for vector $\boldsymbol{x}=[x_1,\cdots,x_n]^{\mathrm{T}}$ is given by

$$\tanh(\boldsymbol{x}):=[\tanh(x_1),\cdots,\tanh(x_n)]^{\mathrm{T}}$$

Proposition 1 can be extended into a vector form as below.

Proposition 2. Consider the vector form of system (2):

$$\begin{cases} \dot{\boldsymbol{\xi}}_1 &= \boldsymbol{\xi}_2 \\ \dot{\boldsymbol{\xi}}_2 &= -\alpha\mathrm{tanh}(k\boldsymbol{\xi}_1+l\boldsymbol{\xi}_2)-\beta\mathrm{tanh}(l\boldsymbol{\xi}_2) \end{cases} \tag{3}$$

where $\boldsymbol{\xi}_1\in\mathbf{R}^n$ and $\boldsymbol{\xi}_2\in\mathbf{R}^n$. The non-trivial solution of system (3) is globally asymptotically stable and semi-globally exponentially stable for any positive numbers α, β, k and l.

Proof of Proposition 2 is presented in Appendix.

Suppose that system (3) is perturbed by a bounded disturbance $\boldsymbol{\Delta}$ with $\|\boldsymbol{\Delta}\|<\bar\Delta$:

$$\begin{cases} \dot{\boldsymbol{\xi}}_1 = \boldsymbol{\xi}_2 \\ \dot{\boldsymbol{\xi}}_2 = -\alpha\mathrm{tanh}(k\boldsymbol{\xi}_1+l\boldsymbol{\xi}_2)-\beta\mathrm{tanh}(l\boldsymbol{\xi}_2)+\boldsymbol{\Delta} \end{cases} \tag{4}$$

The stability property of system (4) can be given in the following proposition.

Proposition 3. Consider the perturbed system (4) with $\boldsymbol{\xi}_1\in\mathbf{R}^n$ and $\boldsymbol{\xi}_2\in\mathbf{R}^n$. Let the expected region of attraction be

$$\left\{[\boldsymbol{\xi}_1^{\mathrm{T}},\boldsymbol{\xi}_2^{\mathrm{T}}]^{\mathrm{T}}\,\middle|\,\left\|[k\boldsymbol{\xi}_1^{\mathrm{T}}+l\boldsymbol{\xi}_2^{\mathrm{T}},l\boldsymbol{\xi}_2^{\mathrm{T}}]\right\|<\bar\mu,\bar\mu>0\right\}$$

Then, there exists $\bar\Delta>0$, such that for $\|\boldsymbol{\Delta}\|<\bar\Delta$ and positive numbers α, β, k and l, the non-trivial solution of (4) is ultimately bounded.

Proof of Proposition 3 is presented in Appendix.

Remark 1. Under the corresponding definitions in some previous literature[24], the result in Proposition 3 is named local input-to-state stability (LISS) with respect to the perturbation $\boldsymbol{\Delta}$.

2 Problem statement

2.1 Mathematical modeling for a model-scaled helicopter

In this paper, the mathematical model of a model-scaled helicopter presented in our previous research[25] is employed. A simple structure of the model-scaled helicopter is illustrated by Fig. 1 where the two reference frames are defined for mathematical modeling.

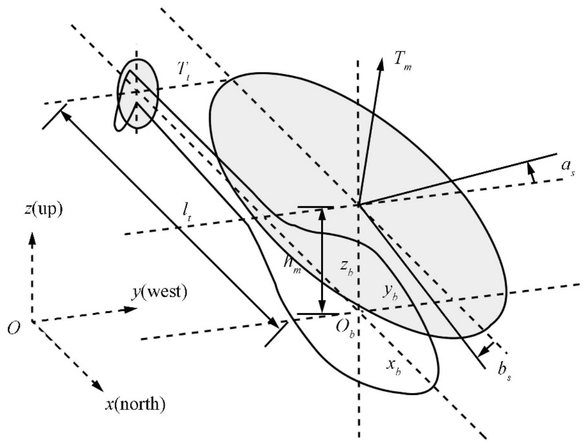

Fig. 1 A simple illustration of the model-scaled helicopter: reference frames, rotor thrusts and flapping angles

The earth reference frame (ERF) is fixed to the earth, with the origin locating at a fix point on the ground. The x axis points to the north and the z axis points upright. The y axis can be confirmed by the right-hand rule.

The fuselage reference frame (FRF) is fixed to the helicopter fuselage. The origin locates at the c.g. (center of gravity) of the helicopter's fuselage, with the x_b axis pointing to the head of the helicopter. The z_b axis is perpendicular to the x_b axis and points upright. The y_b axis can be confirmed by the right-hand rule.

The mathematical model of the model-scaled helicopter could be derived from the Newton-Euler equations[2, 4]:

$$\dot{\boldsymbol{p}}=\boldsymbol{v} \tag{5}$$

$$m\dot{\boldsymbol{v}}=-m\boldsymbol{g}_3+R(\boldsymbol{\gamma})\boldsymbol{f} \tag{6}$$

$$\dot R(\boldsymbol{\gamma})=R(\boldsymbol{\gamma})S(\boldsymbol{\omega}) \tag{7}$$

$$J\dot{\boldsymbol{\omega}}=-S(\boldsymbol{\omega})J\boldsymbol{\omega}+\boldsymbol{\tau} \tag{8}$$

where $\boldsymbol{p}:=[x,y,z]^{\mathrm{T}}$ and $\boldsymbol{v}:=[u,v,w]^{\mathrm{T}}$ are position and velocity of the c.g. of the helicopter in ERF, respectively; m denotes the gross mass; $\boldsymbol{g}_3:=[0,0,g]^{\mathrm{T}}$, and g is the gravitational acceleration; $\boldsymbol{\gamma}:=[\phi,\theta,\psi]^{\mathrm{T}}$ denotes the attitude of the fuselage; the rotational matrix is given by

$$R=[R_{ij}]:=\begin{bmatrix} \mathrm{c}\theta\mathrm{c}\psi & \mathrm{c}\psi\mathrm{s}\theta\mathrm{s}\phi-\mathrm{c}\phi\mathrm{s}\psi & \mathrm{c}\phi\mathrm{c}\psi\mathrm{s}\theta+\mathrm{s}\phi\mathrm{s}\psi \\ \mathrm{c}\theta\mathrm{s}\psi & \mathrm{s}\psi\mathrm{s}\theta\mathrm{s}\phi+\mathrm{c}\phi\mathrm{c}\psi & \mathrm{c}\phi\mathrm{s}\psi\mathrm{s}\theta-\mathrm{s}\phi\mathrm{c}\psi \\ -\mathrm{s}\theta & \mathrm{c}\theta\mathrm{s}\phi & \mathrm{c}\theta\mathrm{c}\phi \end{bmatrix}$$

where $\mathrm{c}(\cdot)$ and $\mathrm{s}(\cdot)$ stand for $\cos(\cdot)$ and $\sin(\cdot)$, respectively; $\boldsymbol{\omega}:=[p,q,r]^{\mathrm{T}}$ represents the angular velocity in FRF; $S(\cdot)$ denotes the skew-symmetric matrix such that $S(\boldsymbol{\omega})J\boldsymbol{\omega}=\boldsymbol{\omega}\times J\boldsymbol{\omega}$; the inertial matrix is given by

$$J:=\begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix}$$