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Online Minimum-acceleration Trajectory Planning with the Kinematic Constraints

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Abstract A novel approach based on a type of simplified motion planning (SMP) is presented in this paper to generate online trajectory for manipulator systems with multiple degrees of freedom (DOFs). The key issue is to find minimum-acceleration trajectory planning (MATP) to optimize the arm motion to reduce disturbance. Moreover, necessary and sufficient conditions for solution′s existence subject to all the kinematic constraints of joint position, velocity, acceleration and jerk are devised. Besides, this new method can be activated online from the arbitrary initial state to the arbitrary target state so that it enables the robot to change the original path at any time. Finally, the approach is applied to a real humanoid robot arm with seven DOFs to show its efficiency.

Key words Motion control, manipulation planning, minimum-acceleration control, simplified motion planning (SMP)

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It is well known that the trajectory planning is a fundamental issue in robotics. A robot′s mechatronics system is limited by the size of its actuators, so that it cannot produce large accelerations and velocities[1−2]. Meanwhile, the robot manipulators are coupled, multivariable, highly nonlinear systems which make it very difficult to ensure an optimal smooth motion within all the kinematic constraints of velocity, acceleration and jerk[3−7].

A simple alternate solution is transforming motion of the robot in Cartesian space into the joint space. Consequently, the problem of optimal motion planning is divided in two steps[8]: firstly, a sequence of path points are derived, whose number is the tradeoff between computational expense and exactness, and which is usually specified in terms of a desired position and orientation of the tool frame in Cartesian space; secondly, each of these points is mapped into a set of joint angles by the application of the inverse kinematics, and then these sets of joint angles are interpolated with smooth functions to be optimized, subject to constraints accordingly chosen for a specific robot application.

Typically, optimization is achieved by minimizing a suitable performance index while satisfying appropriate smoothness requirement for the task. For the mobile manipulator, due to the dynamics coupling between the arm and the mobile base (such as humanoid, spacecraft)[9−10], the movement of the robot manipulator could not only disturb the balance and stability of the mobile body, but also affect its performance. Corresponding to the reactive force against base, the acceleration of the arm′s movement would essentially disturb mobile body, especially when the arm fixed on the spacecraft is in free-floating situation. Therefore, in this paper, the acceleration of the arm in mobile manipulator has been assumed as performance index for the mobile manipulator task.

Further, in order to obtain the online minimum-acceleration trajectory in the robot′s joint-space within an assigned time while taking into account all given kinematic constraints, a kind of acceleration planning approach named simplified motion planning (SMP) is introduced, then the sufficient and necessary condition of the solution′s existence is devised. SMP is from the arbitrary initial state to the arbitrary target state (i.e. the arbitrary velocity and acceleration) under the kinematic constraints. Moreover, the completeness of the proposed condition is proven, like, if the trajectory meets all the requirements, the corresponding solution will be achieved, otherwise, the reason that which kinematic constraint cannot be satisfied, will be given. Based on the SMP, for a given target joint position which meets the proposed condition, the final joint trajectory will be obtained. The trajectory generated by the proposed approach has been proven to have minimum-acceleration profile and limited jerk for keeping the balance and reducing its impact on the base. The method can be universally applied and is not only useful for a robot′s joint space (both revolute joints and prismatic joints), but also useful for any other trajectory planning problems.

The rest of the paper is organized as follows: Section 1 introduces the related works. Section 2 describes the main problems of minimum-acceleration trajectory planning (MATP) and the notations all over this paper. Section 3 proposes MATP from the arbitrary initial state to the arbitrary target state in a determined time with all the kinematic constraints. Section 4 presents the experimental results applied to a real humanoid robot arm with seven DOFs.

1 Related works

The problems of trajectory planning in robotics have been intensively studied for many years. References [11−15] are all about the time-optimal trajectory planning, whose main problem is to find the shortest-time trajectory from an initial state to a target state with some kinematic constraints. Macfarlane and Croft[11] used a concatenation of quantic polynomials to provide a smooth and jerk-bounded trajectory between two points, which was near time-optimal for the jerk and acceleration limits specified; Kim et al.[12] solved a minimum-time trajectory planning problem based on the combined dynamic model of a mobile robot; Behzadipour and Khajepour[13] searched the trajectory planning of high-speed cable-based parallel manipulators for a given geometrical path; the classic seven-segment acceleration profile was used in [14] to get the time-optimal result, but the velocity and acceleration of the target both can be zero; Kröger[15] provided the time-optimal trajectory planning for multi-axes robot for the arbitrary initial

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and target states.

If the motion time is determined, an interesting approach is to minimize the jerk. Several methods have been studied for this issue[16−22], whose main problem was to find the minimum-jerk trajectory from an initial state to a target state with a determined time. Piazzi and Visioli[16] presented a new approach to find the global minimum-jerk cubic spline joint trajectory of a robot manipulator using interval analysis, which was an offline approach that planned the minimum-jerk trajectory by neglecting constraints on velocity and acceleration; Huang et al.[17] used a genetic algorithm to search the optimal joint inter-knot parameters in order to realize the minimum jerk; Kyriakopoulos and Saridis[18] presented a simple method of trajectory generation based on an optimal control problem formulation, where the concept of the minimum energy problem in joint space and jerk minimization in Cartesian space were both introduced; the optimal smooth piecewise trajectories in 3D environments was proposed in [19], where the concept of minimum energy was introduced as well; fifth-order B-splines were used in [20] to ensure that the resulting trajectory is smooth enough and the squared jerk was regarded as an optimal objective function.

Beyond the mentioned works, seldom work deals with the arbitrary state (i.e. the velocity and acceleration are not equal to zero, at this time the problem will become more complex), which is important for online obstacle avoidance[23] or modifying the original path instantaneously. Moreover, all proposed approaches are aimed to find the minimum jerk or squared jerk for the fixed base, there are hardly any studies on finding the minimum-acceleration trajectory, which is also an important factor in the mobile manipulator systems for keeping the balance and reducing its disturbance.

2 Problem description and notations

In this section, we propose the main problem of MATP that will be solved in this paper, and give the first impression how the trajectory is mathematically represented and introduce some notations throughout this paper.

2.1 Problem description

Assume that the initial state of one joint is denoted by $\{q_0, v_0, a_0\}$, and the target state of the joint is denoted by $\{q_f, v_f, a_f\}$, where the parameters q, v, a represent the joint position, velocity, and acceleration, respectively. The kinematic constraints are expressed as $\{v_{\max}, a_{\max}, j_{\max}\}$, which represent the boundaries of velocity, acceleration, jerk, respectively. The main problem of MATP is to find a trajectory from the initial state $\{q_0, v_0, a_0\}$ to the target state $\{q_f, v_f, a_f\}$ within determined time t_f, which is continuous in acceleration and satisfies all the kinematic constraints. Simultaneously, from all that satisfies the requirements above, we need to find the one that is acceleration-optimal, namely, the one whose maximum value of acceleration of the trajectory is minimum.

MATP could be described as follows:

Suppose that the function $a(t)$ represents the acceleration profile of the trajectory, then the corresponding velocity and joint position functions can be represented:

$$v(t) = v_0 + \int_0^t a(s)\mathrm{d}s \tag{1}$$

$$q(t) = q_0 + \int_0^t v(s)\mathrm{d}s \tag{2}$$

So that the motion planning can be denoted by $a(t)$, $v(t)$ or $q(t)$, the main problem of MATP contains two aspects:

1) To confirm whether there exists a continuous acceleration profile $a(t)$, which is subject to (3) ∼ (7):

$$a(0) = a_0, a(t_f) = a_f \tag{3}$$

$$v(0) = v_0, v(t_f) = v_f \tag{4}$$

$$q(0) = q_0, q(t_f) = q_f \tag{5}$$

$$\left|\frac{a(t_1) - a(t_2)}{t_1 - t_2}\right| \le j_{\max}, |t_1 - t_2| < \varepsilon, \forall t_1, t_2 \in [0, t_f] \tag{6}$$

$$|v(t)| \le v_{\max}, \forall t \in [0, t_f] \tag{7}$$

2) Suppose all the acceleration profiles that satisfy all the requirements form a set Ω_a, MATP could be expressed by a minimax optimization problem:

$$\min_{a(t)\in\Omega_a} \max\{|a(t)|, t \in [0, t_f]\} \tag{8}$$

2.2 Notations

We use Fig. 1 to explain all kinds of notations. All over this paper, unless noted otherwise, $a_{adef}(t)$ is represented as the acceleration function of the open curve $ADEF$, and $v_{adef}(t)$ and $q_{adef}(t)$ as the velocity and joint position, respectively. Δv_{adef} and Δq_{adef} denote the increment of the velocity and joint position in the whole motion process:

$$\begin{aligned}\Delta v_{adef} &= v_{adef}(t_f) - v_{adef}(0)\\ \Delta q_{adef} &= q_{adef}(t_f) - q_{adef}(0)\end{aligned}$$

where the subscript $adef$ are the corresponding lower-case letters of all the via points in the curve $ADEF$ (there maybe any number of via points between A and F). Besides, for the arbitrary point (for example, the point B) in the figure, we use (t_b, a_b) to denote its coordinate, and we use S_{acfb} to denote the area of the close curve $ACFB$.

3 MATP algorithms

In this section, the problem of MATP will be solved, at first, we find the trajectory subject to (3), (4) and (6); second, we find the minimum-acceleration trajectory subject to (3) ∼ (6); then we cope with the constraint of velocity to make the trajectory subject to (3) ∼ (7), and summarize the whole process to find the the minimum-acceleration trajectory satisfying all the requirements; finally, the problem of joint limit is discussed in detail.

3.1 Simplified motion planning

First, we find the trajectory subject to (3), (4) and (6), which means SMP. Explicitly, the problem of SMP is to find a trajectory from the state $\{v_0, a_0\}$ to the target state $\{v_f, a_f\}$ within a determined time t_f with limited jerk, while we do not need to consider the joint position and the constraint of velocity temporarily.

As shown in Fig. 1, there is an acceleration profile, where the gradient of the straight line AC and FB are both $j_{\max}$, the gradient of the straight line AB and CF are both $-j_{\max}$. The corresponding acceleration function of each straight line is shown in Table 1.