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A model for the jitter of a ring oscillator under sinusoidal interference on the power supply

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ABSTRACT

This paper presents a model with analytical expressions for the jitter of a ring oscillator (RO) caused by single-tone sinusoidal interference on the power supply. The value of the jitter can be calculated given the static delay of the inverter and the size of the RO. The model explains the relationship between the jitter and the interference parameters, such as frequency and amplitude. Transistor-level simulations and test-board measurements are performed. The jitter calculated from the model corresponds well with the experimental results over a wide frequency range. The positions of the local maxima and minima in the frequency response are precisely calculated by the model. The jitter model is applicable even under large-amplitude interference. The unusual negative-growth behavior in the amplitude response predicted by the model is also observed in the experimental results. The traditional linear theory for the jitter of a ring oscillator is based on the assumption of a small signal. The model in this paper is valid for more general cases, as it can be used to explain and predict key features of jitter.

1. Introduction

The ring oscillator (RO) is a widely used functional part of modern electronic systems. Recently, ROs have been proposed to replace phase-locked loops as the main on-chip clock course due to the ability of ROs to track the process, voltage and temperature variability of the chip [1]. As a clock source, the jitter property of the RO is an important concern.

ROs suffer from jitter due to internal and external interference. The fluctuations in the power supply caused by such interference is a main cause of the jitter. The relationship between the jitter and the waveform of the supply noise should be investigated to understand the operation of ROs.

Sinusoidal waveforms are an important class of variation. Research on the jitter of ROs under sinusoidal variation on the supply can be traced back to the 1990s [2]. studied the cycle jitter and cycle-to-cycle jitter of an RO due to radio frequency interference (RFI) in the supply. A linear model was used for low supply noise. The jitter was proposed to be proportional to the interference amplitude in the supply [3]. stated that the supply noise influences the phase of the RO through a multi-node injection mechanism, but the brief discussion did not provide a detailed model for noise waveform and jitter [4]. assumed that the propagation speed in a delay line, which can be considered an opened ring of gates, is proportional to the supply voltage, resulting in a formula in which the maximal variation in the propagation time of a delay line is a periodic function of f_{RFI}/f_{clk} [5]. presented a delay model for power-supply noise of arbitrary waveform in which the gate delay is determined by the average value of the waveform [2–4]. treated the jitter using linear models, which are not suitable for analyzing large-amplitude electromagnetic interference (EMI) [5]. considered only the average noise, which is not suitable for sinusoidal interference because the average value of sinusoidal interference is zero. Recently [6,7], presented an alpha model for jitter. The model is in principle suitable for large-amplitude EMI, but it is only at the behavioral level and does not describe the mechanism and characteristics of the jitter.

Numerous other publications have focused on the jitter of ROs. Several studies have focused on jitter due to intrinsic noise, such as thermal noise and flicker noise. These studies focused on the spectrum of the oscillation signal. Other studies focused on the dynamics of the injection locking of the RO. Those works do not analyze the same physical scenario as this paper; the physical scenario considered here is shown in Fig. 1. Recent updates on these topics can be found in Refs. [1,8–12].

This paper aimed to create a new jitter model that explains the origin of the jitter caused by supply noise. First, the gate delay of the inverter is considered, and the jitter of the RO is derived. Characteristics such as the

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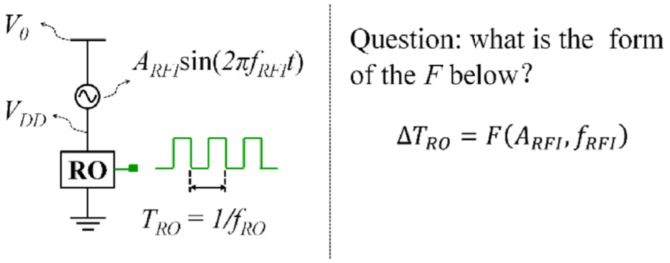

Fig. 1. Physical scenario analyzed in this paper.

frequency response and amplitude response of the jitter are quantitively reflected and explained. HSPICE simulation is performed to verify the theory. In addition, ROs are built on a test board, and on-board measurements are conducted. Finally, the conclusions of this study are presented.

2. Theory for jitter under interference

2.1. Definition of jitter

An RO is a closed loop of cascaded inverters. Under static operation, each stage has the same delay. The oscillation period of the RO with N_{RO} inverters is given by

$$T_{RO} = \sum_{k=1}^{2N_{RO}} \tau_k = 2N_{RO}\tau_{inv} \tag{1}$$

$$Jitter = \Delta T_{RO} = T_{RO\_max} - T_{RO\_min} \tag{2}$$

The RFI in the supply influences the operation of the RO. Thus, the propagation delay is no longer a simple constant but varies over time. Jitter is introduced to reflect the fluctuation of the RO's oscillation period. Multiple statistic measures are used to quantify jitter, such as the root mean square and standard deviation. In our model, we will use the difference between the largest and smallest values of the measured T_{RO} (i.e., the range) as the metric of jitter, as shown in Equation (2).

In the theoretical analysis, various functions and parameters will be used. Their physical meaning and units are summarized in Table 1. Detailed illustrations are given in the next sub-sections.

The following derivations decomposes the RO cycle time into two parts: a f_{RFI} dependent part and a f_{RFI} independent part. The later one is related to a phase evolution of the interference during the switching of a certain amount of inverter stages. The jitter corresponds to the variation of the phase evolution. Function f_N as well as g_N is built to make conversion between number of the switched stages and the phase evolution of the interference. Only a part of the stages of an RO is important for the jitter. The number of those stages is determined by parameter α_{REM} and Ω_{UC}. The response of the jitter to the interference frequency and amplitude is obtained by examining the relationship between $\{f_{RFI}, A_{RFI}\}$ and $\{f_N, \alpha_{REM}, \Omega_U\}$.

2.2. Jitter due to periodic interference

The static propagation delay τ_{inv} of the inverter depends on the supply voltage V_{DD}:

$$\tau_{inv} = \tau_{inv}(V_{DD}) > 0 \tag{3}$$

n_{sw} is defined as the number of switched inverters per unit time:

$$n_{sw} = \frac{1}{\tau_{inv}(V_{DD})} \tag{4}$$

When interference is injected to V_{DD}, V_{DD} will fluctuate over time, which implies that inverters in the RO might be switched under different V_{DD}. The waveform of V_{DD} can be partitioned several sub-regions, as shown in Fig. 2. Sub-region i has its own values of V_{DD} and V_i and a time interval Δt_i.

Table 1
Functions and parameters.

Symbol	Physical Meaning	Unit
T_{RO}	Cycle time of an RO	s
ΔT_{RO}	Jitter of an RO	s
τ_{inv}	Delay of an inverter stage	s
A_{RFI}	Interference amplitude	V
f_{RFI}	Interference frequency	Hz
T_{RFI}	Interference cycle time	s
N_{RO}	Stages of inverters in an RO	n.a.
N_{TRFI}	Number of the switched stages within one entire interference cycle	n.a.
N_{REM}	Number of the switched stages within the last (a faction) interference cycle	n.a.
φ	Phase of the interference	rad
θ	End value of φ in a time interval	rad
$f_N(\theta)$	(1) A function independent from f_{RFI} (2) Together with f_{RFI} determines the number of the switched stages as φ evolves from 0 to θ	rad/s
Ω_{UC}	Together with f_{RFI} determines the number of the switched stages as φ evolves from 0 to 2π (an interference cycle)	rad/s
$g_N(\Omega)$	(1) Inverse function of f_N (2) Helps find the θ for certain switched stages	rad
Ω	Argument of function g_N, equivalent to f_N	rad/s
α_{REM}	The ratio between N_{REM} and N_{TRFI}, normalized value	n.a.
α_0	α_{REM} at present A_{RFI}	n.a.
$\Delta\alpha$	Increment in α_{REM} when A_{RFI} is increased	n.a.
ψ	Evolution of φ for the switching of N_{REM} stages	rad
$\Delta\psi$	Variation of ψ, together with f_{RFI} determines ΔT_{RO}	rad
a, b, c	Coefficients for calculating the gate delay	s, Vs, V

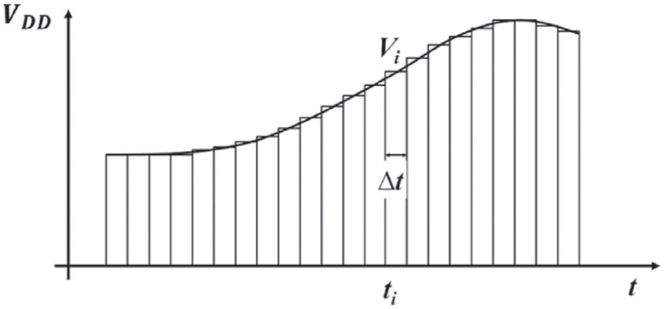

Fig. 2. Partitioning of the time-varying VDD waveform.

$$T_{RO} = \sum_{i=1}^{M} \Delta t_i \tag{5}$$

Suppose that one cycle of RO oscillation covers M sub-regions; then, the sum of the intervals of the M sub-regions yields T_{RO}, as shown in (5), and the sum of switched stages yields $2N_{RO}$, as shown in (6).

$$2N_{RO} = \sum_{i=1}^{M} n_{sw}(V_i)\Delta t_i = \sum_{i=1}^{M} \frac{\Delta t_i}{\tau_{inv}(V_i)} \tag{6}$$

As Δt approaches zero, an integral with respect to time t can be obtained, as shown in (7).

$$2N_{RO} = \lim_{\Delta t\to 0} \sum_{i=1}^{M} \frac{\Delta t_i}{\tau_{inv}(V_i)} = \int_0^{T_{RO}} \frac{1}{\tau_{inv}(V_{DD}(t))} dt \tag{7}$$

RFI can be treated as a type of periodic interference. Its phase is φ_0, as shown in (8). The supply voltage is the sum of the nominal voltage V_0 and a function $v_T(\varphi)$ with period of 2π, as shown in (9). Parameters A_{RFI} and ω_{RFI} represent the amplitude and angular frequency of the interference, respectively.