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Application of expectation maximization and Kalman smoothing for prognosis of lumen maintenance life for light emitting diodes

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A B S T R A C T

Light emitting diodes (LED) have found widespread use for lighting and displays in recent times due to their high efficiency, favorable form factor, impact resistance, robustness, reliability and prolonged lifetime. The extended times to failure of LEDs make the problem of lumen maintenance life prediction for LED light sources using traditional reliability-based failure data collection and analysis methods quite challenging. Therefore, one has to resort to using prognostic approaches such as Kalman or particle filters applied to real time degradation data to make inferences on the remaining useful life (RUL) of such devices. The standard prognostic approach for predicting lumen maintenance life relies on an underlying degradation model with predetermined parameters from previous stress tests. However, these model parameters may vary quite a bit from device to device and for different manufacturing batch lots, thereby leading to large prediction errors. To address this issue, it would be better to learn the model parameters from the current measurement data directly. This study aims to achieve this by estimating the parameter values using the Expectation Maximization (EM) algorithm. Kalman smoothing is then applied to the identified parameter values for predicting the lumen maintenance life of the LED. The accuracy of the EM – Kalman smoothing approach was tested and compared with the standard nonlinear least square (NLS) approach prescribed by the TM-21 standard as well as the standard particle filter approach (without EM). Our results show that the EM method can give better, if not, similar RUL prediction accuracy with respect to the NLS and standard particle filter (PF) algorithms. Moreover, the accuracy of RUL prediction using our proposed algorithm is insensitive to the choice of the initial model parameter values, which paves way for this algorithm to be used in practice for automated predictive analytics of degrading electromechanical systems.

1. Introduction

Light-emitting diodes (LEDs) are popularly used in recent times for indoor lighting, street lamps, communication, medical devices, advertising displays, electronic gadget screens, decorative lighting and much more [1]. LED light sources are known for their high efficiency, environmental resilience, mechanical toughness, high reliability and long lifetime, with claims of the mean time to failure extending to 50,000 h or longer [2–4]. However, owing to their inherent prolonged reliability [5,6], it becomes expensive and time consuming for LED developers to predict the life for LEDs using traditional accelerated run-to-failure life testing, which provide us with the statistical distribution of the time to failure data. Moreover, accelerated test results only provide an overall perspective of the population to estimate the mean and variance in the lifetime distribution, but do not provide specific lifetime information on any particular LED under study. This necessitates the use of prognostics and health management (PHM) techniques for real-time or offline prediction of the remaining useful life (RUL) of the LED based on past and current time-based gradual degradation of its luminous intensity.

Most PHM algorithms for micro-optoelectronic devices start with an underlying degradation model, which can be physical/phenomenological or empirical depending on our depth of knowledge of the failure physics of the device under study. One of the common models prescribed for LED lumen degradation is the single exponential model [7,8], $LM\% = B\exp(-\alpha t)$, where LM refers to the lumen maintenance and $\{B, \alpha\}$ are the model parameters to be estimated. Wang and Lu [9] later extended this model to a bi-exponential model for better representation of the lumen degradation trends in LEDs and Cai *et al.* used a composite exponential model for the same [10]. The choice of the exponential model was purely empirical here based on the generic trend of the data observed. Other than the exponential family of models, the lumen degradation prediction has also been accomplished

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in the past using a fully physics based rate kinetics theoretical model [11], Wiener process [12], Brownian motion process [13] and Gamma process [14,15].

The generic approach to estimate the model parameters of the degradation models above has been to adopt the non-linear least square (NLS) method, which is also recommended by the TM-21 standard (refer to Appendix A for more details). However, it is apparent from several studies by Fan *et al.* [16–18] that the NLS method suffers from poor prediction accuracy as it does not account for the perturbations inherent in the measurement and uncertainties in the model and the operating conditions. As such, the use of NLS is not a preferred option for RUL prognosis in most practical scenarios.

As a better alternative, particle filter-based (PF-based) approaches [19–21] have become more popular for prognosis of lumen degradation in LEDs as they are able to handle non-linear degradation trends as well as non-Gaussian noise perturbations. Particle filter is a well-known general nonlinear filtering method [22]. It is simple and easy to implement and applicable to a wide range of degradation scenarios. However, the downside of the PF method is that it requires initialization of the parameters by non-linear curve fitting, as shown by Fan *et al.* in Ref. [18], where five out of nine devices under test (DUTs) were used as training samples for initializing the parameter values for the PF method. In the event of the initial parameters being considerably off from their true values due to lack of prior knowledge, the sequential Monte Carlo procedure yields erroneous or very widely distributed results for the predicted RUL. It is therefore essential to have a robust approach to parameter initialization before the PF algorithm or any of its variants can be used for actual prognosis. The other issue with the PF is its computational load which makes it less suitable for on-line prognosis in real-time.

This paper examines the use of the Expectation-Maximization (EM) algorithm for estimating the parameters of the degradation model directly from the data, thereby bypassing the initialization step. The same data from a DUT can be used for both parameter value estimation and remaining useful life prediction of lumen degradation. We also show in this paper that the simple linear filtering method (Kalman-based) may be sufficient enough for predicting lumen maintenance life of LEDs (due to its pseudo-linear degradation trends) with the same accuracy and much lower computational load. The estimated parameter values from our EM algorithm will be used within the Kalman filtering (linearized) framework for prediction of lumen degradation and RUL estimation.

2. Model framework for parameter estimation and prognosis

In prognosis and remaining useful life (RUL) prediction, it is normally assumed that the parameter values of the state-space model are reasonably well known and only the future states need to be estimated. However, in practical cases, the parameter values are unknown as well. In this section, we first describe the essence of the Expectation Maximization (EM) algorithm for parameter estimation of a linear state space system. Subsequently, we will show how the outcome from the EM algorithm can be applied for parameter estimation and prediction of lumen degradation trend and eventual failure.

2.1. Parameter estimation for linear state space model by expectation-maximization (EM) algorithm

Let us consider the estimation of vector parameters $\boldsymbol{\theta}$ from the linear state space model:

$$\begin{aligned}\mathbf{x}_k &= A(\theta)\mathbf{x}_{k-1} + \mathbf{q}_{k-1}\\ \mathbf{y}_k &= H(\theta)\mathbf{x}_k + \mathbf{r}_k\end{aligned} \tag{1}$$

where $\mathbf{q}_{k-1} \sim N(\mathbf{0}, Q(\boldsymbol{\theta}))$, $\mathbf{r}_k \sim N(0, R(\boldsymbol{\theta}))$ are Gaussian process and measurement noise, $\mathbf{x}_k$ is the system state and $\mathbf{y}_k$ represents the measurement data. The problem to be addressed here is to find an estimate for $\widehat{\theta}$ from T measurements, $\{\mathbf{y}_1, \ldots, \mathbf{y}_T\}$.

The model in Eq. (1) can be described in a probabilistic form as follows:

$$\begin{aligned}&\theta \sim p(\theta)\\ &\mathbf{x}_0 \sim N(m_0(\theta), P_0(\theta))\\ &\mathbf{x}_k \sim p(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \theta)\\ &\mathbf{y}_k \sim p(\mathbf{y}_k \mid \mathbf{x}_k, \theta)\end{aligned} \tag{2}$$

where $\mathbf{x}_0$ denotes the initial condition and $\{m_0(\boldsymbol{\theta}), P_0(\boldsymbol{\theta})\}$ are the initial mean and covariance.

The full posterior joint distribution can then be formed by the Bayesian rule:

$$p(\mathbf{x}_{0:T}, \theta \mid \mathbf{y}_{1:T}) = \frac{p(\mathbf{y}_{1:T} \mid \mathbf{x}_{0:T}, \theta)p(\mathbf{x}_{0:T} \mid \theta)p(\theta)}{p(\mathbf{y}_{1:T})} \tag{3}$$

where

$$\begin{aligned}p(\mathbf{x}_{0:T} \mid \theta) &= p(\mathbf{x}_0 \mid \theta)\prod_{k=1}^{T} p(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \theta);\\ p(\mathbf{y}_{1:T} \mid \mathbf{x}_{0:T}, \theta) &= \prod_{k=1}^{T} p(\mathbf{y}_k \mid \mathbf{x}_k, \theta)\end{aligned} \tag{4}$$

The full posterior density can then be integrated with respect to the states to yield a marginal posterior of the parameter:

$$p(\theta \mid \mathbf{y}_{1:T}) = \int p(\mathbf{x}_{0:T}, \theta \mid \mathbf{y}_{1:T})d\mathbf{x}_{0:T} \tag{5}$$

The maximum likelihood estimation of the parameters can then be found by

$$\widehat{\theta} = \max_{\theta}(p(\theta \mid \mathbf{y}_{1:T})) \tag{6}$$

However, this formulation requires the computation of a high-dimensional integral in Eq. (5). The EM method relies on recursively computing an approximation of Eq. (5) as given by:

$$p(\theta \mid \mathbf{y}_{1:T}) \propto p(\mathbf{y}_{1:T} \mid \theta)p(\theta) \tag{7}$$

The marginal likelihood, $p(\mathbf{y}_{1:T} \mid \theta) = \prod_{k=1}^{T} p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1}, \theta)$, can be computed recursively as [23]:

$$p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1}, \theta) = \int p(\mathbf{y}_k \mid \mathbf{x}_k, \theta)p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}, \theta)d\mathbf{x}_k \tag{8}$$

where $p(\mathbf{y}_k| \mathbf{x}_k, \boldsymbol{\theta})$ is the measurement model and $p(\mathbf{x}_k| \mathbf{y}_{1:k-1}, \boldsymbol{\theta})$ is the predictive distribution which satisfies the relationship:

$$\begin{aligned}p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}, \theta) &= \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1}, \theta)p(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1}, \theta)d\mathbf{x}_{k-1}\\ p(\mathbf{x}_k \mid \mathbf{y}_{1:k}, \theta) &= \frac{p(\mathbf{y}_k \mid \mathbf{x}_k, \theta)p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}, \theta)}{p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1}, \theta)}\end{aligned} \tag{9}$$

Therefore, the problem in Eq. (6) can now be solved by minimizing the negative log-likelihood:

$$\widehat{\theta} = \min_{\theta}[-L_{\theta}(\mathbf{y}_{1:T})] = \min_{\theta} -\left(\log p(\theta) + \sum_{k=1}^{T} \log(p(\mathbf{y}_k \mid \mathbf{y}_{k-1}, \theta))\right) \tag{10}$$

The objective function in Eq. (10) can be minimized using the *gradient method*. However, the gradient method may not guarantee an increase in the value of $L_{\boldsymbol{\theta}}(\mathbf{y}_{1:T})$. In contrast to the gradient based estimation method, the *EM algorithm* is an iterative procedure, whereby the l^{th} step seeks an estimate, $\boldsymbol{\theta}^{(l)}$, that guarantees $L_{\boldsymbol{\theta}^{(l)}}(\mathbf{y}_{1:T}) > L_{\boldsymbol{\theta}^{(l-1)}}(\mathbf{y}_{1:T})$. The main idea of the EM approach is to consider the joint likelihood function with respect to both measurement and the states, $L_{\boldsymbol{\theta}}(\mathbf{y}_{1:T}, \mathbf{x}_{1:T})$. Since $\mathbf{x}_{1:T} = \{\mathbf{x}_1, \ldots, \mathbf{x}_T\}$ is not available, the value for $L_{\boldsymbol{\theta}}(\mathbf{y}_{1:T}, \mathbf{x}_{1:T})$ is approximated by using the minimum variance estimation, $\varphi(\boldsymbol{\theta}, \boldsymbol{\theta}^{(l)})$, of the joint likelihood function $L_{\boldsymbol{\theta}}(\mathbf{y}_{1:T}, \mathbf{x}_{1:T})$, where