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A generalized degradation model based on Gaussian process

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ABSTRACT

Degradation analysis has been recognized as an effective means for reliability assessment of complex systems and highly reliable products because few or even no failures are expected during their life span. To further our studies on degradation analysis, a generalized Gaussian process method is proposed to model degradation procedures. A one-stage maximum likelihood method is constructed for parameter estimation. Approximated forms for median life and failure time distribution (FTD) percentile are also derived considering the concept of First Hitting Time (FHT). To illustrate the performance of the proposed method, a comprehensive simulation study is conducted. Furthermore, the proposed method is illustrated and verified via two real applications including fatigue crack growth of 2017-T4 aluminum alloy and light emitting diode (LED) deterioration. The Wiener process model with mixed effects is considered as a reference method to investigate the generality of depicting common degradation processes. Meanwhile, to show the effectiveness of considering the FHT concept, another method (that has same model parameters with the proposed approach while does not consider the FHT definition) is also adopted as a reference. Comparisons show that the proposed methodology can not only show significant advantages for time- decreasing dispersity situations where the Wiener process models cannot reasonably perform, but also guarantee an enhanced precision for time-increasing variance circumstances.

1. Introduction

Referring system reliability and safety, failure prediction is always an important issue in both theoretical and practical researches. Considering conventional methods, abundant failure data has to be collected through life tests for a reasonable assessment [1]. For long-life and highly reliable systems which are more and more common in real applications, however, the conventional approaches encounter enormous challenges regarding reliability assessment. The reason lies in that failure lifetime can hardly be collected in a reasonable (usually condensed) life test time span that is necessary for effective and low-cost product developments [2].

In the past two decades, degradation information has been recognized as a rich source for reliability analysis [3]. It can provide a superior alternative to estimate reliability for highly reliable products, because analysts do not have to wait for failures to occur, and more useful information, other than one failure lifetime, can be collected per unit. By defining failure as the degradation reaching a predefined threshold, the relationship between reliability index and performance feature can be established through degradation analysis. This methodology suggests that the actual time to failure may never be observed, but it can be determined by estimating the degradation path and the process dispersion property [4].

Recently, performance degradation reliability analysis has attracted numerous attention from researchers of both theoretical and practical engineering studies [5]. Many models have been developed and applied [6,7]. Among them, Wiener process approach is a well-known model that has been studied from various fields. The reason lies in its excellent mathematical properties and physical interpretations [8]. More references could be found in Whitmore [9] and Yu and Tseng [10]. According to Whitmore and Schenkelberg [11] and Wang [12], a regular Wiener process $\{X(t), t \geq 0\}$ with drift β and diffusion σ can be expressed as a well- adopted form

$$X(t) = \beta\Lambda(t) + \sigma\mathbf{B}(\Lambda(t)) \tag{1}$$

where $\mathbf{B}(\cdot)$ denotes a standard Brownian motion, and $\Lambda(t)$ is a transformed time scale. From the definition of Wiener process, we know the increments of $\{X(t), t \geq 0\}$ are mutually independent and normally distributed; viz. $\Delta X(t) = X(t + \Delta t) - X(t)$ is s-independent of $X(t)$, and $X(t) \sim N(\beta\Lambda(t), \sigma^2\Lambda(t))$.

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In practical modeling procedure, the time-transformed scale form of $\Lambda(t)$ has to be determined first. This time transformed form can be obtained by engineering experience (see the log-transformation based on Paris Law [13]), mechanistic knowledge (see the wearing theory in Wang and Dan [14]), or data plotting (see the complex logarithmic transformation in Zuo et al. [15], the exponential transformed degradation path of light emitting diode (LED) products [16] and the generalized cumulative damage approach introduced by Park and Padgett to describe initial damage [17]). Then by setting different form of $\Lambda(t)$, model of Eq. (1) can properly describe practical degradation sequences of different situations.

To properly consider the unit-specific properties which have been commonly recognized from a practical viewpoint, mixed effects have been further introduced into the Wiener process model [8]. In real applications, drift β for individual unit denoting deterioration rate is usually assumed to be fixed but unknown, and follows a normal distribution $\beta \sim N(\mu_\beta, k^2)$.

To construct a more generalized Wiener process model, Tseng [18] introduced two different transformed time scales $\Lambda(t)$ and $\tau(t)$, and then model of Eq. (1) can be developed as

$$X(t) = \beta\Lambda(t) + \sigma\mathbf{B}(\tau(t)) \tag{2}$$

However, since it is hard to construct a closed-form PDF, only the situation $\Lambda(t) = \tau(t)$ has been studied in the literature regarding the degradation model of Eq. (2).

According to the definition, Wiener process shows an independent increment property, and Wiener processes have to illustrate monotone increasing dispersions because of the positive increments variance. Consequently, the previously discussed Wiener process degradation models can only depict diffusion procedures, but cannot properly model the degradation datasets with decreasing variance, which do exist in real applications. See the LED degradation inspections collected by Chaluvadi [19]. From this viewpoint, the main objective of this study is to construct a more useful and generalized degradation method, which can reasonably analyze practical degradation processes in both increasing and decreasing dispersity situations.

Encompassing Wiener process as a limiting case, Gaussian process is a more generalized stochastic procedure that has been developed since 1940s [20,21]. A universal Kriging method was built for spatial inference in geostatistics [22,23]. Then Gaussian process methodology was further applied in spatial prediction [24] and spatial statistics [25]. Since 1990s, Gaussian process regression has been widely developed and applied in machine learning [26,27]. Regarding degradation reliability analysis, however, few literature has been reported to the best of the authors knowledge. Padgett and Tomlinson [28] defined the continuous damage increment as a function of Gaussian process, but they actually adopted Wiener process in the modeling procedure. Doksum and Normand [29] developed a Gaussian process to describe the relationship between biomarker process values at random time points, but they limited their study to the stationary Gaussian process situation. Therefore, this general process defined as Gaussian process is utilized to develop a more useful degradation model in this paper.

Based on the study conducted by Meeker and Escobar [30], Bagdonavicius and Nikulin [31], Yang and Xue [32], it can be concluded that a model with linear mean and quadratic variance can fit most of the degradation. It is worth noticing that the linear mean and quadratic variance can be constructed directly or via a proper transformation, such as the above discussed methods via engineering experience [13], mechanistic knowledge [14] or data plotting [15]. From this viewpoint, a linear mean and a quadratic variance is considered in the current study.

Considering the randomness of the degradation processes, FHT definition has been usually adopted for a reasonable FTD construction. It is well-known that FHT of a linear Wiener process can be established in a closed-form as an inverse Gaussian distribution [12,33–35]. Some nonlinear deterioration processes can be depicted by linear Wiener process model through proper transformations, and then FHT of these procedures can be derived accordingly [9,36,37]. For more commonly encountered general nonlinear Wiener processes, however, FHT is extremely hard to obtain. Si [38] developed an analytical approximates for FHT of nonlinear Wiener procedures under a mild assumption. Along this viewpoint, FTD for the proposed Gaussian process degradation method is obtained adopting FHT definition.

The rest of this paper is organized as follows. Section 2 introduces the generalized Gaussian process and the degradation model. Section 3 illustrates the modeling procedure. A comprehensive simulation study is given in Section 4 for model validation via comparison. Section 5 involves two real applications to illustrate the performance of the proposed method from a practical viewpoint. Finally a summary and conclusion is given in Section 6.

2. Model introduction

2.1. Gaussian process

A time continuous stochastic process $\{X(t)\}$ is Gaussian distributed if and only if $\boldsymbol{X} = \{X(t_1), X(t_2), \cdots, X(t_n)\}'$ is a multivariate Gaussian random variable for every finite set of indices $t_1, t_2, \cdots, t_n$ [39]. From a theoretical viewpoint, a Gaussian process can be completely defined and depicted by its second-order statistics including mean and covariance [40].

According to GP definition, for a time sequence $t_1, t_2, \ldots, t_n$, the probability density function (PDF) of multivariate Gaussian distributed $\boldsymbol{X} = \{X(t_1), X(t_2), \cdots, X(t_n)\}'$ can be written as

$$f(\boldsymbol{X}) = \frac{1}{(2\pi)^{n/2}\det(\boldsymbol{\Sigma})^{1/2}}\exp\{-(\boldsymbol{X}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}-\boldsymbol{\mu})/2\} \tag{3}$$

where $\boldsymbol{\mu} = \{\mu(t_1), \mu(t_2), \cdots, \mu(t_n)\}'$ denotes the mean vector with $\mu(t_i) = \mathrm{E}(X(t_i))$, and $\boldsymbol{\Sigma} = (\Sigma_{ij})_{n\times n}$ is the covariance matrix with $\Sigma_{ij} = \mathrm{Cov}[X(t_i), X(t_j)]$, $i, j = 1, 2, \cdots, n$.

2.2. Forms of mean and covariance functions

Wiener process (a specific situation) is taken as an example to investigate the relation between the degradation process and the mean and covariance functions. As a specific GP, a Wiener process is essentially a diffusion procedure that shows a monotone increasing dispersion.

For a regular Wiener process depicted by Eq. (1) that has been widely investigated in the literature, it can be concluded that the transformed time scale $\Lambda(t)$ has to monotonously increase with time. This is because the properties of independent increment and diffusion. Therefore, the mean $\mu(t)$ and variance element Σ_{ij} can be expressed as

$$\mu(t) = \beta\Lambda(t) \tag{4}$$

$$\Sigma_{ij} = \sigma^2\Lambda(t_i) \qquad i < j, \qquad i, j = 1, 2, \cdots, n \tag{5}$$

when drift coefficient β and diffusion coefficient σ are both constants.

From the regular Wiener process definition (depicted by Eq. (1)) and statistical characteristic discussions (see Eqs. (4) and (5)), one can conclude that this regular model can only grasp an increasing dispersity. Considering the real deterioration procedures illustrating decreasing variances (see the LED light degradation inspections in Ref. [16] for example), a more generalized degradation model based on Gaussian process is very necessary.

For a general GP $\{X(t)\}$ with a PDF expressed by Eq. (3), its mean $\mu(t) = \mathrm{E}(X(t))$ is a common time-varying function which does not need to monotonously increase with time. More important, the covariance Σ_{ij} of $X(t_i)$ and $X(t_j)$ has also been developed as

$$\Sigma_{ij} = \min\{\mathrm{Var}(X(t_i)), \mathrm{Var}(X(t_j))\} \qquad i, j = 1, 2, \cdots, n \tag{6}$$

From Eq. (6), it can be seen that the covariance Σ_{ij} of $X(t_i)$ and $X(t_j)$