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Stable reduced order modeling of piezoelectric energy harvesting modules using implicit Schur complement

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ABSTRACT

In this work, we present an improved technique for reduced order modeling of a micro-machined piezoelectric energy harvester, presented in Kudryavtsev et al. (2015), and a novel frequency-tunable piezoelectric energy harvester with segmented electrodes for improved power generation. A computationally efficient implicit Schur complement is developed for better conditioning of the numerical models. In combination with Krylov subspace-based model order reduction methods, it offers an efficient way to generate guaranteed stable reduced order models of multi-physical device models. We demonstrate an excellent match between the full-scale and the reduced order models of the harvester devices, and establish a new methodology for system-level simulation based on these reduced order numerical models.

1. Introduction

With the increasing deployment of perpetually powered systems, energy harvesting techniques have attracted considerable research efforts. Several micro-structured harvester devices have been introduced to utilize thermal, solar or vibration energy available in the environment and transform them into electric power [2-6]. Devices harvesting vibration energy often use the piezoelectric effect. Several designs have been proposed so far [7-10]. Since the overall performance of the energy harvesting system is not solely dependent on the harvester design itself, but also on the power management circuit [11-14], system-level simulation is inevitable.

Commonly, system-level simulation integrates a lumped element description of the respective device [15-18]. Even though, this description can give a fair assessment of the energy harvester device, it is difficult to perform an optimization at system-level. Our work supports the use of more accurate, numerical models within a system-level simulation.

In [1], the authors exploit the mathematical framework of model order reduction (MOR) to provide compact models for system-level simulations. A starting point can be a finite element model, which contains a large number of degrees of freedom. MOR decreases this number significantly and thereby enables efficient system-level simulation while maintaining the accuracy of the finite element model.

However, one often encounters stability issues when reducing coupled multi-physics models with state-of-the-art MOR methods [19-21]. In order to overcome this issue, the authors in [1] introduced a procedure called *'MOR after Schur'* and observed that the stability of the model is preserved through the MOR process, provided the Schur complement is performed beforehand.

In this work, we give a first-time proof that *'MOR after Schur'* mathematically preserves stability when applied to piezoelectric energy harvester models, and we extend *'MOR after Schur'* to what we call *'MOR after implicit Schur'*, oriented on the work in [22]. We show that our new method produces the same reduced order model (ROM) as *'MOR after Schur'* and therefore preserves stability, while being significantly more efficient. For the investigation, we consider two case studies: the micro-structured piezoelectric energy harvester introduced in [1] and a novel piezoelectric harvester device with frequency tunability.

Section 2 briefly introduces the two energy harvester devices we considered in our case studies. First, we recapture the micro-structure device, which was the scope of investigation in [1] and then introduce the aforementioned device with frequency tunability.

In Section 3, we introduce some mathematical preliminaries of MOR before we rephrase the *'MOR after Schur'* procedure and prove its stability preservation. Finally, we introduce the newly developed *'MOR after implicit Schur'* and show that it delivers the same ROM.

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Section 4 contains numerical simulation results. We show that the ROM of the frequency-tunable harvester, obtained by our procedures, also accurately replicates the input/output behavior of the original system, and that the computation time is significantly reduced by *'MOR after implicit Schur'*. Furthermore, we show results of a device-circuit co-simulation of the ROM together with a simple bridge rectifier circuit.

Finally, Section 5 concludes the paper and gives an outlook on possible future works.

2. Piezoelectric energy harvester

Vibration-based energy harvesting utilizing the piezoelectric effect is particularly well suited for industrial applications, which employ sensors in inaccessible or harsh environments. Most vibration-based harvesters are spring-mass-damper systems, which generate maximum power when the resonance frequency of the resonator coincides with the ambient vibration frequency. However, in the majority of practical cases, the ambient vibrations are varying in dominant frequency or even show random spectra. Consequently, a mismatch between the vibration and harvester frequency leads to inefficient power generation. This constitutes a fundamental limitation for conventional vibration harvesters. To overcome this drawback, a system of two coupled mass-spring resonators was introduced [23]. In this section, we briefly recapture this micro-structured piezoelectric energy harvester device (Fig. 1). Furthermore, we introduce a new piezoelectric energy harvester with frequency tuning capabilities (Fig. 2).

2.1. Micro-structured piezoelectric energy harvester

The micro-structured piezoelectric energy harvester consists of one inner and two outer mass elements, referred to as *mass* and *trusses* respectively. The mass, the trusses and the surrounding frame are connected via flexible beams. These beams carry piezoelectric patches. Deflection of the beams generates surface strain, which is transformed into a voltage by the piezoelectric patches. The mechanical setup of multiple masses coupled via flexible beams enables the harvester device to operate at two resonant frequencies.

The mechanical part of the micro-structured harvester is entirely made of silicon using techniques introduced in [24]. The piezoelectric patches are fabricated from aluminum nitride (AlN) as the piezoelectric material with an aluminum top and a platinum bottom electrode.

2.2. Frequency-tunable piezoelectric energy harvester

The tunable piezoelectric energy harvester consists of a folded beam in clamped-free configuration (as shown in Fig. 2). Two identical (outer) arms stretch from the base. Their ends are mechanically connected. From that connection a second (inner) beam extends back towards the base. Both beams carry piezoelectric patches that convert mechanical to electrical energy. Two fundamental modes are presented in Fig. 2.

The piezoelectric patches (PIC-255, supplied by PI Ceramic GmbH) are integrated with dimensions of $60 \times 10 \times 0.2\,\text{mm}^3$ and $48 \times 18 \times 0.2\,\text{mm}^3$ for the outer and inner patches, respectively. They consist of thin cupronickel films as top and bottom electrode material, and a piezoelectric ceramic layer with a thickness of 0.2 mm in between. The piezoelectric properties of the ceramic film are given in Tables 1–3. Mechanical material properties are specified in Table 4.

In the present case, the resonator consists of stainless steel with 1 mm thickness. Two masses of identical weight (5 g) are attached to the beams' ends. A harmonic base excitation acts perpendicular to the plane of the device. The resonator significantly amplifies the base displacement amplitudes when the frequency of the excitation coincides with one of the resonance frequencies of the structure. This leads to deformation of the flexible parts of the structure. Depending on the resonance frequency either the outer beams or the inner beam experience substantial deformation. This leads to a polarization of the piezoelectric patches, which is noticeable as a voltage across the patch electrodes.

In contrast to most energy harvester designs, our approach integrates a tuning scheme for adjusting the resonance frequency. This is achieved by external magnets, which exert forces on permanent magnets attached to the flexible structure. These forces superimpose to the the restoring force of the beam; thereby hardening or softening the structure. This altered effective stiffness leads to the desirable frequency shift.

2.3. Finite element model of piezoelectric energy harvester

Finite element models of both harvester designs have been implemented using ANSYS™ (V18.2).

The mathematical description of the systems corresponds to second-order multiple-input multiple-output (MIMO) dynamical systems of the form:

$$\begin{cases} \underbrace{\begin{bmatrix} M_{11} & 0 \\ 0 & 0 \end{bmatrix}}_{M} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix}}_{E} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}}_{K} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}_{B} u, \\ y = \underbrace{\begin{bmatrix} C_1 & C_2 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases} \tag{1}$$

where $M = M^T, E = E^T \in \mathbb{R}^{(n+k)\times(n+k)}$ are the mechanical mass and damping matrices. $K = K^T \in \mathbb{R}^{(n+k)\times(n+k)}$ consists of $K_{11} \in \mathbb{R}^{n\times n}$ describing the structural stiffness, $K_{12} \in \mathbb{R}^{n\times k}$ and $K_{21} \in \mathbb{R}^{k\times n}$ describing the piezoelectric coupling, and $K_{22} \in \mathbb{R}^{k\times k}$ describing the dielectric conductivity. $x_1 \in \mathbb{C}^n$ representing nodal displacement and $x_2 \in \mathbb{C}^k$ representing electrical potentials are parts of the state vector, $u \in \mathbb{R}^l$ is the input vector and $y \in \mathbb{C}^m$ is the user defined output vector, with B being the input and C being the gathering matrix.

The input of both models is the mechanical displacement of the frame respectively base (***displ***). The micro-structured piezoelectric energy harvester model has three displacement defined as outputs: ***centre***, ***south*** and ***north***, corresponding to the displacements of mass and truss elements. The tunable piezoelectric harvester has two mechanical outputs: ***south*** and ***north*** relating to the displacements of the inner and outer mass. Both models provide two electric ports named ***el1*** and ***el2***,

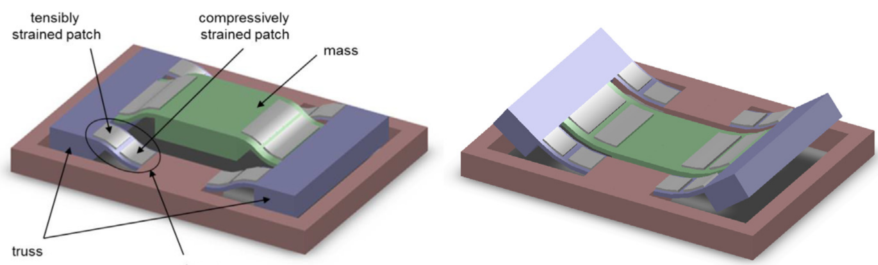

Fig. 1. Descriptive scheme of the micro-structured piezoelectric energy harvester from [1]. Device dimensions are $14 \times 9 \times 0.7\,\text{mm}^3$. Beam thickness is 50 µm.