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Partition-based approach to parametric dynamic compact thermal modeling

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ABSTRACT

This paper presents three procedures for the extraction of parametric Dynamic Compact Thermal Models (DCTMs) with controlled and user-chosen accuracy, namely, (i) a DCTM with dense matrices obtained by a *direct* conventional method, (ii) a *partition-based* approach leading to a sparse DCTM suited for heat conduction problems suffering from a massive number of independent heat sources and/or parameters, for which extracting conventional dense DCTMs may be too resource-demanding or even unviable, and (iii) a novel algorithm that quickly translates a sparse DCTM into a dense one, which allows reducing the simulation time. The proposed methodologies are validated through the application to two state-of-the-art electronics systems.

1. Introduction

During the last decades, various approaches have been proposed in literature for constructing Dynamic Compact Thermal Models (DCTMs) of electronic components and packages [1–4]. In this context, the Authors have proposed effective Model-Order Reduction (MOR) methods for extracting DCTMs for the detailed heat conduction models of electronic components and packages through the Multi-Point Moment Matching (MPMM) algorithm [5–8] and its successive improvements [9–21]. MOR-based DCTMs are an extremely efficient alternative to standard numerical simulations since they allow quickly and accurately evaluating both the key temperatures (point or surface and/or volumetric average) directly as output, and the whole space-time temperature evolution with a subsequent projection [17].

Prominent among the aforementioned models are the *parametric* DCTMs, which explicitly include the dependence on a subset of boundary conditions, material thermal properties, and geometry details, thereby enabling the fast study of the influence of parameter variations in a wide range [11,15,19-21]. In this case, the heat conduction problem to be reduced is preliminarily reformulated in an equivalent way by considering a reference domain with *fixed* geometry and variable material properties (i.e., a single mesh has to be constructed). Bases obtained with a suitable expansion are then introduced for approximating the temperature field, and the discretized equations describing the parametric heat conduction are projected onto the space spanned by the previously-computed bases.

A conventional parametric MOR technique operates on the whole spatial domain describing the electronic component/system, directly extracting *dense* (full) DCTMs [11,15]. In the case of low number of independent heat sources (IHSs) and parameters, the achieved DCTMs have small dimensions and can be numerically solved at negligible computational cost. However, if the number of IHSs and/or parameters is high, the dimension of the bases can become large. This leads to the following drawbacks: the number of matrix elements defining the dense DCTMs *quadratically* increases with the dimension of the bases, thus reducing its advantages in terms of both extraction and – to a lesser extent – simulation times; in severe cases, the orthonormalization of the bases and the projection of the discretized equations onto the space spanned by these bases may even become unviable due to RAM shortage.

In [20], a novel MOR approach was proposed for overcoming the above drawbacks for the case of linear parametric DCTMs, while

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keeping the attractive features of the original technique. In this approach, differently from the aforementioned one, the computational domain is partitioned into sub-regions, each including a small number of IHSs and parameters. The sub-regions are then separately modeled by DCTMs, which together form a DCTM for the whole system characterized by *sparse* matrices instead of the usual dense ones. In addition, the number of nonzero elements of the sparse matrices *linearly* increases with the dimension of the bases [20]. On the other hand, the dimension of sparse DCTMs is larger than that of dense counterparts; this leads to simulation times longer than the ones required by dense DCTMs, unless optimized algorithms are used. As an illustrative example, dense DCTMs are to be preferred for the case of the solution to coupled electrothermal (ET) problems in SPICE [16,18], since a general-purpose code is exploited. A more efficient thermal model is of utmost importance for improving convergence and CPU time of ET analyses, hence the need arises for obtaining dense DCTMs starting from their sparse counterparts.

In [21], improving [20], it is shown how DCTMs defined by sparse matrices can be exploited for deriving DCTMs defined by dense matrices, overcoming the problems of RAM shortage occurring when trying to directly obtain dense models by performing the orthonormalization of the bases and the projection of the discretized equations onto the space spanned by the bases.

This paper extends [20,21] by detailing a procedure for extracting both dense and sparse parametric DCTMs while controlling the approximation error; as a consequence, the DCTM accuracy can be arbitrarily chosen *a-priori* by the user, even for a large number of IHSs and/or parameters. Moreover, for the first time, an approach is presented and discussed to perform a fast sparse-to-dense conversion providing a dense variant of the sparse DCTM that allows reducing the simulation time.

The proposed method can be applied to any discretized form of the heat conduction equations, e.g., obtained by the Finite Element Method (FEM), the Finite Volume Method (FVM), or Finite Integration Technique (FIT) [22]. It does not require that the heat conduction equations are discretized by any partition-based method, such as Domain Decomposition or Discontinuous Galerkin's approaches, thus allowing for easy integration with a desired commercial tool resorting to a given reference numerical method.

The remainder of the paper is articulated as follows. Section 2 describes the general framework of parametric heat conduction problems, illustrating how a DCTM can be defined in terms of port variables. The dense and sparse DCTM extraction procedures guaranteeing a given approximation error are illustrated in Sections 3 and 4, respectively, while the sparse-to-dense conversion is shown in Section 5. Section 6 is devoted to the algorithms validation, which is performed for two state-of-the-art case studies, namely, an IGBT module, and an output stage of a gallium arsenide (GaAs) power amplifier (PA), exhibiting different numbers of IHSs and parameters. As main results, typical DCTM extraction and simulation times are reported. The case studies witness that a significant order reduction can be achieved by adopting the DCTMs, and that the converted sparse-to-dense DCTM features the same characteristics of a DCTM directly extracted in the dense form in terms of order and number of nonzero elements. Conclusions are finally drawn in Section 7.

2. Parametric heat conduction problems

Let a transient heat conduction problem be considered, in which either the material thermal properties or the geometry details can depend on parameters $p_1, \ldots, p_p$, forming vector $\mathbf{p}$. Straightforwardly extending [11] from the steady-state to the transient case, the transient heat conduction problem can be reformulated as an equivalent problem in which *only* thermal properties vary with $\mathbf{p}$, whereas the spatial domain Ω does *not* vary with $\mathbf{p}$. The temperature rise $\vartheta(\mathbf{r}, t, \mathbf{p})$ with respect to ambient temperature T_0, at each position vector $\mathbf{r}$ in Ω and at each time instant t, is ruled by the transient heat diffusion equation

$$c(\mathbf{r},\mathbf{p})\frac{\partial \vartheta}{\partial t}(\mathbf{r},t,\mathbf{p}) + \nabla\cdot(-\mathbf{k}(\mathbf{r},\mathbf{p})\nabla\vartheta(\mathbf{r},t,\mathbf{p})) = q(\mathbf{r},t,\mathbf{p}),$$

in which the heat capacity $c(\mathbf{r},\mathbf{p})$, the thermal conductivity $\mathbf{k}(\mathbf{r},\mathbf{p})$, and the power density $q(\mathbf{r},t,\mathbf{p})$ all depend on $\mathbf{p}$. Such equation is completed by conditions on the boundary $\partial\Omega$, of outward unit normal vector $\mathbf{n}(\mathbf{r})$, here assumed to be homogeneous of Robin's type

$$-\mathbf{n}(\mathbf{r})\cdot\mathbf{k}(\mathbf{r},\mathbf{p})\nabla\vartheta(\mathbf{r},t,\mathbf{p}) = h(\mathbf{r},\mathbf{p})\vartheta(\mathbf{r},t,\mathbf{p}),$$

in which the heat transfer coefficient (HTC) $h(\mathbf{r},\mathbf{p})$ can depend on $\mathbf{p}$, and by an initial condition, which, without loss of generality, is assumed to be homogeneous $\vartheta(\mathbf{r},0,\mathbf{p}) = 0$.

A DCTM is suitably introduced by defining the *thermal ports* [6]. Powers $P_i(t)$, $i = 1, \ldots, n$, injected at the n ports determine the power density in the form

$$q(\mathbf{r},t,\mathbf{p}) = \sum_{i=1}^{n} g_i(\mathbf{r},\mathbf{p})P_i(t),$$

$g_i(\mathbf{r},\mathbf{p})$ being the shape functions, dependent on $\mathbf{p}$, scaled in such a way that

$$\int_\Omega g_i(\mathbf{r},\mathbf{p})d\mathbf{r} = 1.$$

Following [6], the n port temperature rises $\Delta T_i(t,\mathbf{p})$, $i = 1, \ldots, n$, are given by

$$\Delta T_i(t,\mathbf{p}) = \int_\Omega g_i(\mathbf{r},\mathbf{p})\vartheta(\mathbf{r},t,\mathbf{p})d\mathbf{r}.$$

As it can always be assumed in practice [11], the material thermal properties vary with $\mathbf{p}$ as follows:

$$c(\mathbf{r},\mathbf{p}) = \sum_{j=1}^{C} C_j(\mathbf{p})c_j(\mathbf{r}),$$

$$\mathbf{k}(\mathbf{r},\mathbf{p}) = \sum_{j=1}^{K} K_j(\mathbf{p})\mathbf{k}_j(\mathbf{r}), \quad h(\mathbf{r},\mathbf{p}) = \sum_{j=1}^{H} H_j(\mathbf{p})h_j(\mathbf{r}),$$

$$g_i(\mathbf{r},\mathbf{p}) = \sum_{j=1}^{G} G_j(\mathbf{p})g_{ij}(\mathbf{r}),$$

in which $C_j(\mathbf{p})$, $K_j(\mathbf{p})$, $H_j(\mathbf{p})$, and $G_j(\mathbf{p})$ are functions of $\mathbf{p}$ and $c_j(\mathbf{r})$, $\mathbf{k}_j(\mathbf{r})$, $h_j(\mathbf{r})$, and $g_{ij}(\mathbf{r})$ are functions of the position vector $\mathbf{r}$.

As it is well known, only in exceptional situations, in which geometries and material parameters are over-simplified, the transient heat conduction equations can be solved in closed form. Hence, transient heat conduction is usually solved with a numerical method such as FEM, FVM or FIT. Using any of these discretization methods, the heat diffusion equation can be put in the form

$$\mathbf{M}(\mathbf{p})\frac{d\boldsymbol{\vartheta}}{dt}(t,\mathbf{p}) + (\mathbf{K}(\mathbf{p}) + \mathbf{H}(\mathbf{p}))\boldsymbol{\vartheta}(t,\mathbf{p}) = \mathbf{q}(t,\mathbf{p}), \tag{1}$$

in which the unknown $\boldsymbol{\vartheta}(t,\mathbf{p})$ is the columns vector with the M degrees of freedom (DoFs) of the temperature rise with respect to ambient temperature, $\mathbf{M}(\mathbf{p})$ is the mass matrix, and $\mathbf{K}(\mathbf{p})$, $\mathbf{H}(\mathbf{p})$ are the two terms of the stiffness matrix, modeling the internal and the

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