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Optimization of convectively cooled heat sinks

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ABSTRACT

Many factors of heat sink, such as its size and mass, component locations, number of fins, and fan power affect heat transfer. Owing to the opposite effects of these factors on heat sink maximum temperature, we have now a multi-objective optimization problem. A typical optimization case consists of hundreds of heat sink temperature field evaluations, which would be impractical to do with CFD. Instead, we propose to combine analytical results of convection and numerical solution of conduction to address these so-called conjugated heat transfer problems. We solve heat conduction in a solid numerically using the finite volume method and tackle convection with the analytical equation of forced convection in a parallel plate channel.
This model is suitable for forced and natural convection heat sinks, and we have verified its validity by comparing its results to measured data and CFD calculations. We use the model to improve two industrial examples, using a multi-objective version of the particle swarm optimization (PSO) algorithm. The first example is a forced convection heat sink composed of nine heat generating components at the base plate, and the other is a natural convection case with two components. In both cases, mass is minimized; the other criterion is maximum temperature for the forced convection case and heat sink outer volume for the natural convection case. Our method is many orders of magnitude faster than CFD. Additionally, we provide some LES results of pin fins with natural convection for further use in similar optimizations.

1. Introduction

A typical heat sink used to cool electronics contains many discrete surface or flush mounted components. An example heat sink containing nine components with different heat dissipation is shown in Fig. 1. It is composed of rectangular plate fins and a base plate. However, the shape is not limited to rectangular, and fins can have another cross-section such as triangular and trapezoidal. The main design criterion is to keep component temperatures below safe values to prevent overheating. If the fins are far apart, a single fin heat transfer analysis is adequate, but this approach is limited in practical applications, which often require calculation of flow and heat transfer in channels between the fins. Isothermal arrays with an optimum channel width have been studied [1], but if we want to minimize mass or size, we must resort to non-isothermal analysis. Because the temperature distributions of a solid and fluid are solved simultaneously, this is a typical conjugated heat transfer problem.

Single fin optimization results do not generally correspond to optimal fin shapes for fin arrays. However, single fin results serve as a good initial guess to optimize the array geometry. Analysis of single fin heat transfer can be found in the literature, e.g., in [2]. The optimum shape of a single plate fin with a constant mass has also been studied in [3], and easy-to-use analytical formulas for optimal fin shapes have been established for forced and natural convection cases when the fin base temperature is constant. Optimum results of fins with other cross-sections such as rectangular also appear in the literature [4]. Recently, Lindstedt and Karvinen arrived at a simple analytical solution for a fin with constant heat flux at the base, a result that helps determine in optimization the minimum fin mass when heat flux and maximum temperature are fixed [5].

Designing a fin array is intrinsically more challenging than single fin case. A fin array problem contains more variables, many of them with opposite effects such as the location of the heat generating components, the number and geometry of the fins, and the outer volume of the array. In addition, the amount of manufacturing material and the power used by pumps or fans can affect the result. In some cases, a somewhat elaborate analytical solution can be used [6], but such multi-objective optimization problems can always be solved numerically [7].

This paper presents, in addition to single fins results, a quick method to calculate the temperature field and heat transfer of a heat sink cooled by forced or natural convection. The method is suitable for multi-objective optimization, and its application is presented in the following. The main idea of the method is that convection heat transfer is solved from analytical equations, and that a numerical solution is used only for conduction. We present two practical examples of how multi-objective optimization can help determine the minimum size and mass of a heat sink for forced and natural convection cooling. First, however, we

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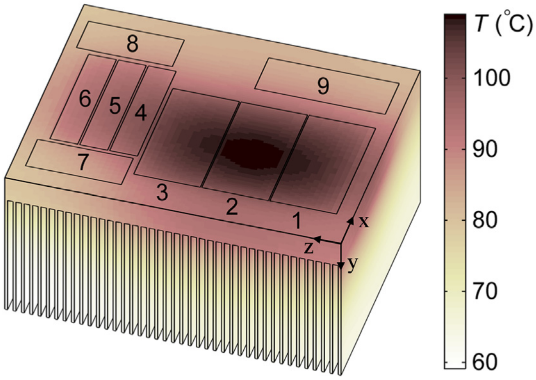

Fig. 1. Schematics and temperature field of an existing heat sink with plate fins and nine components at the base plate (array mass m = 6.65 kg).

present some analytical results of single fins, which serve as a basis for understanding the optimal fin shape and thus help design the first versions of a fin array.

2. Optimal shape of a single fin

The total heat transfer of a fin in Fig. 2 is obtained from the heat transfer of an isothermal fin ϕ_i, which is easily calculated from the correlations of an isothermal plate in text books

$$\phi = \eta\phi_{\mathrm{i}}, \tag{1}$$

if the fin efficiency η is known. The efficiencies of fins with different geometries, which take into account the non-uniform temperature of a fin, are presented for *forced convection* using the non-dimensional variable [3,4]

$$X_* = \frac{1}{C}\frac{kt_0}{k_f l^2}\frac{L}{Re^{m_1}Pr^{n_1}}, \tag{2}$$

where the Reynolds number $\mathrm{Re} = u_\infty L/\upsilon$, Pr is the Prandtl number = 0.7 for gases, υ is the kinematic viscosity, k and k_f are heat conductivities of array material and fluid. Coefficients for a laminar boundary layer are $C = 0.332$, $m_1 = 1/2$, and $n_1 = 1/3$; and for a turbulent boundary layer $C = 0.0296$, $m_1 = 4/5$, and $n_1 = 3/5$. Fig. 2 shows the fin dimensions in Eq. (2). For instance, the efficiency of a rectangular fin in Fig. 2 ($\mathrm{p} = 0$) takes the form

$$\eta = (m_1 X_*)^{1/2}\ \tanh(m_1 X_*)^{-1/2}. \tag{3}$$

The corresponding result of a triangular fin ($\mathrm{p} = 1$) is

$$\eta = (m_1 X_*)^{1/2}\frac{I_0\left(2(m_1 X_*)^{-1/2}\right)}{I_1\left(2(m_1 X_*)^{-1/2}\right)}. \tag{4}$$

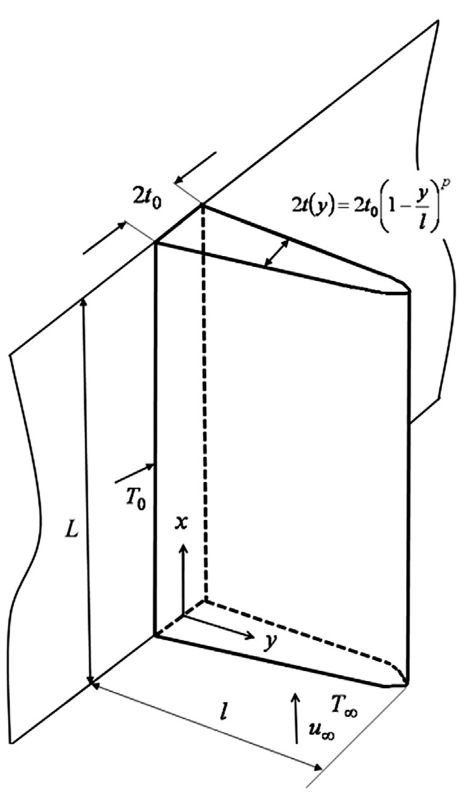

Fig. 2. Geometry of single fin.

In Eq. (4), I_n is the modified Bessel function of the first kind. From Eqs. (3) and (4), it is easy to find the fin geometry to maximize heat transfer for a fixed fin mass when some of the dimensions are fixed, as we often have in actual practice. In Table 1 [4], we give an idea of how flow type and fin shape affect the total heat transfer rate by comparing the relative performance of different fins with the same mass. An optimized aluminum rectangular fin with a fin base thickness of $2t_0 = 1$ mm, and a laminar boundary layer serves as a reference with a heat rate $\phi_{\mathrm{ref}} = 5.1$ W. We can see that the flow type, i.e., laminar or turbulent, has only a little effect on the relative values (1 and 1.18), but that the shape affects more (1 and 1.41) for laminar flow. If the mass of the fin remains the same, changing a laminar boundary layer into to a turbulent one while simultaneously modifying a rectangle shape into a triangle doubles the heat transfer relative value from 1 to 2. If the boundary layer is either laminar of turbulent, the increase in heat transfer is mainly due the increase in L, the fin length in flow direction, while the fin height l stays approximately constant, as seen in Table 1. The optimization thus suggests that there is too much mass located near the fin tip and it ought to be used as increased heat transfer surface area.

In the case of natural convection, the corresponding non-dimensional variable as Eq. (2) is

$$X_* = \frac{1}{C^4}\frac{\nu^2}{g\beta\,Pr}\frac{1}{\theta_0}\left(\frac{kt_0}{k_f l^2}\right)^4 L, \tag{5}$$

Table 1
Dimensions and characteristics of optimal aluminum fins with laminar and turbulent boundary layers in air flow. Fin material volume 10^{-6} m^3 (m = 2.7 g), $u_\infty = 10$ m/s, $\theta_0 = T_0 - T_\infty = 60$ °C, and fixed fin base thickness $2t_0 = 1$ mm.

p laminar	ϕ/ϕ_{ref}, $\phi_{\mathrm{ref}} = 5.1$ W	η	L	l
0	1	0.79	0.051	0.040
1/2	1.23	0.79	0.074	0.040
1	1.41	0.78	0.100	0.040
2	1.52	0.75	0.157	0.038
p turbulent	ϕ/ϕ_{ref}	η	L	l
0	1.18	0.91	0.084	0.024
½	1.61	0.91	0.131	0.023
1	2.00	0.91	0.187	0.021
2	2.63	0.87	0.300	0.020

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