Annual Reviews in Control 35 (2011) 34–55

Contents lists available at ScienceDirect

Annual Reviews in Control

journal homepage: www.elsevier.com/locate/arcontrol

# The input amplitude saturation problem in QFT: A survey

J.C. Moreno [a,*], J.L. Guzmán [a], A. Baños [b], M. Berenguel [a]

[a] Dep. de Lenguajes y Computación, 1.48 CITE III, Universidad de Almería, Ctra de la Playa, s/n, 04120 Almeria, Spain
[b] Dep. de Ingeniería y Sistemas, Universidad de Murcia, 30100, Spain

ARTICLE INFO

*Article history:*
Received 9 September 2010
Accepted 11 February 2011

*Keywords:*
QFT
Robust control
Parametric uncertainty
Saturation

ABSTRACT

In this work the input amplitude saturation problem is analysed in the Quantitative Feedback Theory (QFT) framework. This paper reviews previous works in the literature dealing with the input amplitude saturation problem in the presence of an uncertain plant in the frequency domain using QFT. The objective of this paper is to compare the different available approaches and summarize the design process for each case so that this paper can be used as a tutorial; there are six main approaches to this problem. Two of these approaches use the classical two degrees of freedom control scheme for QFT; in both of these, the design constraints of a linear QFT compensator are added in the loop shaping stage: they are added in the first approach to avoid excitation of the actuator saturation and in the second one to guarantee global stability. The other three techniques are considered as anti-windup (AW) approaches. Starting from a base design in QFT with two degrees of freedom, the first AW approach introduces a third degree of freedom that guarantees the stability of the system, allowing for base designs for high performance. The other two AW approaches also introduce a third degree of freedom, but they take simple stability considerations into account and focus on the performance of the system. The last solution consists of using a reference governor technique, which guarantees the computation of a reference signal for an inner control loop that is shaped using QFT in such a way that robust stability will be guaranteed. The reference governor technique is a time domain approach that implies the resolution of an optimization problem. The rest of the approaches are frequency domain techniques based on a loop shaping method in the traditional QFT sense.

## 1. Introduction

Actuator saturation is a common and significant nonlinearity in practical control systems due to the limited power of the actuators that perform the control action. If this input constraint is not taken into account in the control system design, the performance and stability may be degraded when the controller is implemented. If the controller contains integrators, the well known phenomenon referred to as integral windup may occur: when the control signal saturates, the feedback is broken and the controller continues integrating the tracking error, providing larger control signals and resulting in large overshoots or even driving the system to instability. This phenomenon was first observed in proportional-integral controllers, but as pointed out by Doyle, Smith, and Enns (1987), any controller with relatively slow or unstable modes will experience windup problems if there are actuator constraints. Windup is then interpreted as an inconsistency between the controller output and its state.

During the last three decades, most of research related to control problems due to saturating elements has been focused on the problem of stabilizing LTI plants in the presence of input amplitude constraints. The design of control systems with hard constraints is a very active research area; for example, see (Alamo, Limon, Cepeda, Fiacchini, & Camacho, 2006; Bernstein & Michel, 1995; Grimm, Teel, & Zaccarian, 2004; Marcos, Turner, Bates, & Postlethwaite, 2006; Moreno, Baños, & Berenguel, 2010; Reinelt, 2001; Stoorvogel & Saberi, 1999; Turner, Hermann, & Postlethwaite, 2007; Weston & Postlethwaite, 2000; Wu & Jayasuriya, 2001b) and the references therein. Formal definitions of the anti-windup problem covering $L_p$-stability and performance can be found in (Teel & Kapoor, 1997). However, some of these papers solve this problem in the frequency domain.

In this work, a nonlinear feedback system (with saturation) is described by two modes or states: it is a non-saturated mode when the input and output of the saturating element are equal, and it is a saturated mode when the input and output are different. In addition, if the saturating element is eliminated in the original feedback setup, the resulting LTI feedback system will be referred to as the base LTI system; this base LTI system will be also referred to as the linear mode of the original nonlinear feedback system.

---

* Corresponding author. Tel.: +34 950 015677; fax: +34 950 015129.
*E-mail addresses:* jcmoreno@ual.es (J.C. Moreno), joguzman@ual.es (J.L. Guzmán), abanos@um.es (A. Baños), beren@ual.es (M. Berenguel).

1367-5788/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.arcontrol.2011.03.002

There are three main existing approaches to cope with the saturation problem. The first approach to deal with the problem, commonly referred to as the anti-windup technique, uses a specific controller in order to compensate for the harmful effects caused by the nonlinear element when the control signal is saturated; for example, see (Kothare, Campo, Morari, & Nett, 1994). This is the classical two-step design paradigm: first the linear controller is designed while ignoring the saturation (for the base LTI system), and then an anti-windup compensator is added to minimize the adverse effects of saturation on the closed loop performance. The main advantage of this approach is that the linear and nonlinear modes are decoupled from a design point of view. This linear anti-windup compensator approach is commonly referred to as linear conditioning (Campo & Morari, 1990; Doyle et al., 1987; Edwards & Postlethwaite, 1998; Hippe & Wurmthaler, 1997; Kothare et al., 1994; Weston & Postlethwaite, 2000). In the second approach, the control system is designed in a single step that considers both the linear and nonlinear modes; see, for example, (Tyan & Bernstein, 1995) for a modified anti-windup framework, (Camacho, 1993) for a model predictive control framework, (Alamo et al., 2006) for the framework of robust invariant sets, or (Horowitz, 1983; Horowitz & Liao, 1986) for the framework of quantitative feedback theory (QFT). The problem with this approach is that there is coupling between the linear and nonlinear modes. Thus, the performance of the linear mode is affected by the nonlinear mode compensator and vice-versa, so the design process is much less transparent. Finally, in a third approach, the bounds on the system inputs are assumed and the goal of the design is to guarantee that the control system always operates in the linear mode (as long as the plant and the controller are linear and time-invariant), i.e., the objective is to avoid actuator saturation; see (Herman & Franchek, 2000; Miller & Pachter, 1996; Reinelt, 2001; Reinelt & Canale, 2001) and the references therein. This is a common practice in problems such as automated air traffic management systems (Pappas, 1996; Sastry et al., 1995); however, a drawback of this technique includes a possible lower performance. The common low-gain design technique uses this approach. Since low-gain controllers underutilize the available control capacity, often one finds that the convergence of the error signal to zero as time goes to infinity is rather slow, thus an improved technique called low-high-gain design method may be used. The low-high-gain design method utilizes the available control capacity in a better way, resulting in better performance. See reference (Saberi, Stoorvogel, & Sannuti, 2000) for an overview of these techniques.

Another important aspect in practice is plant uncertainty. A variety of solutions have been proposed in recent years that are often based on LMI techniques. The majority of these methods use an uncertainty representation that may be conservative in many practical cases. Typically, the uncertainty is given as a disturbance (Grimm et al., 2004) or it is described by using a norm (Alamo et al., 2006; Marcos et al., 2006; Turner et al., 2007). In general, robustness and saturation problems have been approached separately (Turner et al., 2007) and there have been few attempts to unify the results and techniques from both research lines.

QFT is especially well suited for dealing with potentially large parametric uncertainty, and it has been shown to be efficient at adapting robust versions of classical global stability, such as Circle, Popov, and multiplier-based criteria (Baños & Barreiro, 2000; Baños, Barreiro, Gordillo, & Aracil, 2002; Barreiro & Baños, 2000). In these works, a robust version of previous criteria is used to introduce new restrictions in the loop using a typical 2 degree of freedom (2-DoF) control scheme $\{F,G\}$. A limitation of this approach is that the LTI base system and the nonlinear mode are coupled during design. In addition, conditionally stable LTI base designs are not allowed. This limitation is overcome in (Moreno et al., 2010), where a new DoF is added to the classical QFT controller in order to assure the absolute stability of the closed loop system, even when a conditionally stable LTI base design is used. Furthermore, Horowitz analysed control systems with saturating elements, but plant uncertainty was only considered for the linear mode (Horowitz, 1983; Horowitz & Liao, 1986).

Other recent related QFT works include references (Herman & Franchek, 2000; Moreno, Baños, & Berenguel, 2003; Oldak, Baril, & Gutman, 1994; Wu & Jayasuriya, 1999, 2001a; Yang, 1992), which develop the Circle Criterion and the Describing Function. In reference (Herman & Franchek, 2000), frequency domain conditions are used to assure the non-excitation of the saturation element and QFT is used to take the uncertainty into account, thereby designing the controller for linear and nonlinear operation in a single step. A frequency domain condition is also given to guarantee a non-saturated steady-state, allowing for a temporary excitation of the saturation element; all of these conditions depend on the amplitude of the input signal. In (Yang, 1992) the problem is studied for sampled-data systems. Assuming a nonlinear element input with limited amplitude, the describing function is used as a variable gain that modifies the plant template and QFT is used in its traditional form. In (Oldak et al., 1994) the effect of the nonlinearity is handled as a disturbance, assuring closed loop stability via the describing function method or the circle criterion. However, this approach is not applicable when the saturation element is located at the plant input. In (Guzman, Alamo, Berenguel, Dormido, & Camacho, 2007, 2009) a reference governor technique is used to avoid actuator saturation by using a QFT controller in the inner loop to reduce the uncertainty that the predictive controller governing the system reference must handle. On the other hand, in (Chan & Hui, 1998), a design method based on the generalized circle criterion is proposed. However, it is assumed that all of the parameters of the plant are exactly known. A tutorial on nonlinear QFT, including a survey of nonlinear control systems with hard nonlinearities (including saturation and backlash), may be found in (Baños, 2007).

The goal of the present work is to summarize and compare the techniques to solve the amplitude saturation problem in a QFT framework. Six approaches are available to deal with this problem. Two of these approaches (Baños & Barreiro, 2000; Barreiro & Baños, 2000; Baños et al., 2002; Herman & Franchek, 2000) introduce design constraints for a 2-DoF linear QFT compensator: no excitation of the actuator saturation is guaranteed for the first technique and global stability is assured for the second one. The other three techniques are considered to be anti-windup (AW) approaches. Starting from a base design in QFT with 2-DoF, one approach (Moreno et al., 2010) introduces a third DoF that guarantees the absolute stability of the system and allows for high performance base designs; and the other two AW approaches (Moreno et al., 2003; Wu & Jayasuriya, 2001a), which are based on the seminal work (Horowitz, 1983), also introduce a third DoF, but take single stability considerations into account and focus more on the system's performance. The sixth approach (Guzman et al., 2007, Guzman, Alamo, Berenguel, Dormido, & Camacho, 2009) uses a reference governor technique that guarantees the computation of a reference signal for the closed inner loop designed using QFT, such that the actuator output is within the saturation limit. The reference governor technique is a combined frequency-time domain approach, implying the resolution of an optimization problem. Other approaches include frequency domain techniques based on a loop shaping method in the traditional QFT sense.

In this paper, the input amplitude saturation problem is analysed in the QFT framework. The structure of this work is as follows. Section 2 briefly introduces QFT and the saturation problem is set. In Section (3), frequency domain based approaches are summarized: techniques using a 2-DoF control structure, which avoid saturation in one case and assure absolute stability