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A stochastic, contingency-based security-constrained optimal power flow for the procurement of energy and distributed reserve☆

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ABSTRACT

It is widely agreed that optimal procurement of reserves, with explicit consideration of system contingencies, can improve reliability and economic efficiency in power systems. With increasing penetration of uncertain generation resources, this optimal allocation is becoming even more crucial. Herein, a problem formulation is developed to solve the day-ahead energy and reserve market allocation and pricing problem that explicitly considers the redispatch set required by the occurrence of contingencies and the corresponding optimal power flow, static and dynamic security constraints. Costs and benefits, including those arising from eventual demand deviation and contingency-originated redispatch and shedding, are weighted by the contingency probabilities, resulting in a scheme that contracts the optimal amount of resources in a stochastic day-ahead procurement setting. Furthermore, the usual assumption that the day-ahead contracted quantities correspond to some base case dispatch is removed, resulting in an optimal procurement as opposed to an optimal dispatch. Inherent in the formulation are mechanisms for rescheduling and pricing dispatch deviations arising from realized demand fluctuations and contingencies. Because the formulation involves a single, one stage, comprehensive mathematical program, the Lagrange multipliers obtained at the solution are consistent with shadow prices and can be used to clear the day-ahead and spot markets of the different commodities involved.

1. Introduction

This work combines several standard problems found in systems operation and planning in a single mathematical programming framework. The advantages of this formulation are found in greater clarity with respect to the underlying problem to be solved, and for ease of extraction of sensitivity information from the solution. The problems herein considered are

- The optimal power flow problem with a full AC nonlinear network model and constraints;
- The $N-1$ contingency security problem with both static (post-contingency voltage and MVA limits) and dynamic (generator ramp rate limits; voltage angle difference limits; post-contingency load pickup governed by participation factors) constraints;
- The problem of procuring an adequate supply of both active and reactive power and corresponding geographically adequate distributed reserves in a day-ahead market scenario in light of the uncertainty of the actual realized demand and the occurrence of specific contingencies, while taking into account the costs and constraints on the corresponding post-contingency flows;
- The problem of setting the price for the day-ahead contracts for power and reserve; and
- A consistent mechanism for re-dispatching and pricing the next day under a specific realization of the set of all uncertain quantities involved.

Each of these problems is usually tackled separately, in a sequential process that revises the original dispatch produced by an optimal power flow solver to accommodate the additional restrictions. However, the sequential nature of typical practice does not ensure that these are introduced in a way that preserves optimality for the overall problem, nor allows for the original LBMPs to be used correctly for pricing both active and reactive power and reserve, or for understanding the price of security. The approach employed here tries to accommodate as many of the issues involved as possible in a single, consistent mathematical program, avoiding the use of proxies of the constraints. The specific novelty in this work lies in 1) the decoupling of the concept of day-ahead programmed dispatch and day-ahead contracted quantity, resulting in an optimal day-ahead hedge for the system operator; 2) a single stage, comprehensive problem formulation for energy and

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Nomenclature

p_{ik}, q_{ik} — ith active and reactive injection in kth post-contingency state ($k = 0$ for base case).
$C_{Pi}(\cdot)$, $C_{Qi}(\cdot)$ — cost function for ith active and reactive injections.
p_{ci}, q_{ci} — purchase amounts specified in the day-ahead contract for active and reactive power from the ith injection.
p^+_{ik}, q^+_{ik} — ith active and reactive upward deviations from contracted amount in kth post-contingency state; $k = 0$ means realized deviation from contract with no contingencies.
$C^+_{Pi}(\cdot)$, $C^+_{Qi}(\cdot)$ — cost for incremental deviations from contract day-ahead quantity.
p^-_{ik}, q^-_{ik} — ith active and reactive downward deviations from contracted amount in kth post-contingency state.
$C^-_{Pi}(\cdot)$, $C^-_{Qi}(\cdot)$ — cost for decremental deviations from contracted day-ahead quantity.
r^+_{Pi}, r^+_{Qi} — upward active and reactive reserve amount provided by ith injection.
$C^+_{RPi}(\cdot)$, $C^+_{RQi}(\cdot)$ — cost functions for upward reserve purchased from ith injection.
r^-_{Pi}, r^-_{Qi} — downward active and reactive reserve amount provided by ith injection.
$C^-_{RPi}(\cdot)$, $C^-_{RQi}(\cdot)$ — cost functions for downward reserve purchased from ith injection.
$(\Theta^k, V^k, P^k, Q^k)$ — voltage angles and magnitudes, active and reactive injections for power flow in kth post-contingency state ($k = 0$ means no contingency occurred).
$g^k(\cdot)$ — nonlinear power flow equations in kth post-contingency state.
$h^k(\cdot)$ — transmission, voltage, generation and other limits in kth post-contingency state.
π_k — probability of kth contingency (π_0 is the probability of no contingency).
n_g — number of generators and dispatchable or curtailable loads initially available.
n_c — number of contingencies considered.
G^k — set of indices of generators present in the kth contingency. Individual variables can be grouped in vectors, such as p_{ik} into P^k, and it will be consistent with the context.

reserve allocation that is appropriate for extraction of sensitivity information important to microeconomics, namely, meaningful location-based shadow prices. The resulting problem is formidable to solve but it exhibits a structure that is amenable to decomposition and coordination approaches to its solution, making a parallel implementation possible and desirable.

Secure operation of generation and transmission systems addresses a plethora of issues. It involves planning so that the system can survive the occurrence of certain kinds of events, most notably so-called “contingencies”, in which a piece of equipment goes offline suddenly. But it also involves planning so that the system can continue to perform if the operating conditions expected at the decision-making moment do not materialize exactly, i.e. if there is uncertainty in the prediction of load, climate, wind or river flow. Of these two types of issues, perhaps the first results in more acute concerns, because a sudden realization of a contingency disturbs the state of the system before much can be done by the operators.

Several events occur in different time frames after a contingency. First, new bus voltages can be reached in a matter of seconds as the transient governed by automatic reactive controls takes place. If the controls steer the voltage towards a stable equilibrium, it still remains to be seen if the overall voltage profile that is reached is appropriate. In a longer time scale involving tens of seconds, frequency controls steer generators to balance the active power and make up for lost generation or load. Under-frequency relays may trigger network reconfiguration events in extreme cases at this stage. In a time frame of a few minutes, area exchange controls balance deviations from scheduled transactions, and operator-originated redispatches start to take place. In some cases, an automatic redispatch is initiated right after the contingency in order to improve the security and economy of the initial post-contingency operating point.

A key planning decision is the amount and location of spinning reserve that must be set aside for eventual use in case of a contingency. The required redispatches might not be feasible otherwise. Thus, correctly solving the planning problem requires addressing the issue of geographically appropriate reserve allocation. Furthermore, correct pricing of this commodity requires that it be explicitly included in the formulation.

A taxonomy of system states with respect to security is offered in [1]. The normal state is that of “secure”, when no operating limits are violated and no limits would be violated in the event of a contingency. Secure operation requires planning with respect to credible contingencies in order to both position the current state accordingly and to plan for corrective rescheduling strategies in the event that one of them does occur. There are many approaches to solving this problem, depending on the formulation, the simplifications, the available tools, and on the numerical method used. Some are only approximate in light of the simplifications, e.g. DC flow instead of AC flow, and require further examination before claiming that the solution is engineering-feasible. Others do not produce accurate pricing information due to the nature of the solution method employed, or the use of proxy constraints instead of precise models of the physical limitations. One key criterion is whether the approach is 1) direct, 2) base flow data modification-based or 3) base flow with added self-contained constraints. The first approach is used, for example, in [2–12] and involves actual simultaneous formulation of the post-contingency flows with additional constraints that bound the deviations of the injections in the post-contingency flows from those in the base case. These are the only coupling constraints; voltage security and rating limits are imposed directly on the post-contingency flows. Clearly, as more contingencies are considered the problem's size becomes formidable and it is tempting to exploit the problem structure with a decomposition framework, typically a price coordination scheme such as Benders' decomposition or Lagrangian relaxation, among others.

The second idea relies on modification of the original problem data for the base case OPF so as not to violate limits in a post-contingency state. A typical example is to artificially reduce the rating in a transmission line or the maximum generation capacity in a given unit to alleviate a congestion problem that would occur in a post-contingency state. This is amenable to sequential modification of a base case OPF after a given OPF solution is analyzed and found to be insecure with respect to contingencies. However, the order in which contingencies are studied might be important in determining the final secure dispatch, which raises the possibility of not finding the true optimum.

The third idea adds more constraints to the base case OPF to force the resulting solution to be secure. Like the second approach, it is amenable to sequential introduction of constraints into the base OPF, dictated by an analysis of the security of a given solution. These new constraints may typically be linearizations of the constraints that were violated in a post-contingency flow, and are thus proxies that may not be entirely accurate.

We now discuss some of the ingredients of the overall problem and how they have been dealt with over the years. Every now and then, reference will be made to specific MATPOWER implementation conventions and algorithms. This stems from the fact that this software package's generalized optimal power flow capabilities have been taken advantage of in order to code the prototype implementation. A detailed description of its capabilities and algorithms can be found in [13,14].

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