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How to preserve the conflict as an alarm in the combination of belief functions?

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ABSTRACT

In the belief function framework, a unique function is induced from the use of a combination rule so allowing to synthesize all the knowledge of the initial belief functions. When information sources are reliable and independent, the conjunctive rule of combination, proposed by Smets, may be used. This rule is equivalent to the Dempster rule without the normalization process. The conjunctive combination provides interesting properties, as the commutativity and the associativity. However, it is characterized by having the empty set, called also the conflict, as an absorbing element. So, when we apply a significant number of conjunctive combinations, the mass assigned to the conflict tends to 1 which makes impossible returning the distinction between the problem arisen during the fusion and the effect due to the absorption power of the empty set. The objective of this paper is then to define a formalism preserving the initial role of the conflict as an alarm signal announcing that there is a kind of disagreement between sources. More exactly, that allows to preserve some conflict, after the fusion by keeping only the part of conflict reflecting the opposition between the belief functions. This approach is based on dissimilarity measures and on a normalization process between belief functions. Our proposed formalism is tested and compared with the conjunctive rule of combination on synthetic belief functions.

1. Introduction

Since many years, the belief function theory [8,43] has known an increasing interest from scientific community since it allows to deal with imperfect data (imprecise and uncertain) and to combine them using a combination rule. One of the classical combination rules is the conjunctive rule. This latter, introduced by Smets [44,48], is equivalent to the Dempster rule of combination [8,43] without the normalization process. Properties and also hypotheses that sources should satisfy before being combined by this rule are well established.

This rule has an orthogonal behavior which is very precious because it permits a fast and clear convergence towards a solution, but in return, the empty set is an absorbing element. Smets supports that the existence of this mass on the empty set, called also conflict, can play a role of alarm. So, contrary to Dempster's rule where the conflict is reallocated proportionally to the other masses of the focal elements, this conflict must not be redistributed since it may be at the origin of important information concerning the progress of the fusion process and show the disagreement between sources. In fact, if the conflict is small, it means that the joint bba fits with the opinions given by the sources to fuse and consequently try to reinforce them, whereas when the conflict is high, it means that the induced bba is largely in contradiction with the previous opinions. Nevertheless, due to its absorbing conjunctive effect, a series of combinations aims at getting the empty set equal to 1, making impossible the distinction between a real problem between sources to fuse and an effect caused by the absorbing of the empty set.

In addition to the conflict definition of Smets, other works have been dealt with the conflict definition namely Liu [30] proposes a quantitative measure taking into account the mass on the empty set induced from the combination of two or more bbas and the distance between betting commitments of these same bbas after applying the pignistic transformation. However, this mass on the empty set remains not sufficient to exactly express the conflict. On the other hand, in [38], Osswald and Martin present another interpretation of the conflict by defining the auto-conflict as the amount of intrinsic conflict of a belief function, in other words it is the conflict generated by such a function relative to one information source.

Besides in [11], Destercke and Burger present some properties that a measure of extrinsic conflict should satisfy. They defined conflict as the inconsistency arising from a conjunctive combination, and based on properties, they also proposed conflict measurements making no a priori assumptions regarding the dependence between sources.

Thus, two types of conflict can be defined:

- The conflict which allows the estimation of the confusion rate of a source and which will be called intrinsic conflict [5,20,38,42],
- The conflict which evaluates the discordance between two bodies of evidence and will be labeled extrinsic conflict [23,39,49].

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In this paper, we characterize the opposition between belief functions by means of a measure of dissimilarity. This measure is then used in our proposed approach named Combination With Adapted Conflict (CWAC) providing an adaptive weighting between Dempster's rule and conjunctive rule, allowing to keep the initial meaning of the conflict obtained during the combination and so to restore its initial role of alarm. Thus, it permits to the conflict to take back its initial sense by only mentioning that there is a problem somewhere and reducing its absorbing power. Our proposal is not a conflict measure but a combination rule preserving the main role of a conflict as a signal making aware of this opposition between sources. A preliminary work of this approach has been proposed in [29].

This paper is organized as follows. Section 2 presents the basics of the belief function theory. Combination rules, proposed in the belief function framework, are detailed in Section 3. The definition and properties of our CWAC rule are exposed in Section 4. Section 5 brings to light our proposed approach by comparing its behavior with that of the conjunctive combination in the case of synthetic data. Section 6 concludes our study and presents some future works.

2. Belief function theory: background

The belief function theory is considered as a useful theory for representing and managing uncertain knowledge. In this Section, we shall briefly recall some basics of this theory. More details can be found in [43,44,48].

2.1. Representing information

Let Ω be a finite non-empty set including all the elementary events related to a given problem. These events are assumed to be exhaustive and mutually exclusive. Such set Ω is named frame of discernment.

The impact of a piece of evidence on the different subsets of the frame of discernment Ω is represented by the so-called basic belief assignment (bba), called initially by Shafer [43] basic probability assignment.

The bba m is a function $m : 2^{\Omega} \rightarrow [0,1]$ that satisfies:

$$\sum_{A \subseteq \Omega} m(A) = 1. \tag{1}$$

The basic belief mass $m(A)$, expresses the part of belief exactly committed to the event A of Ω given a piece of evidence. Due to the lack of information, this quantity cannot be apportioned to any strict subset of A.

Shafer [43] has initially proposed a normality condition expressed by:

$$m(\emptyset) = 0 \tag{2}$$

Such bba is called a normalized basic belief assignment.

Smets [44,45] relaxes this condition by considering $m(\emptyset)$ as the amount of conflict between the pieces of evidence or as the part of belief given to the fact that none of the hypotheses in Ω is true. All the subsets A of Ω such that $m(A)$ is strictly positive, are called the focal elements of m.

Associated with m is the belief function, denoted bel, corresponding to a specific bba m, assigns to every subset A of Ω the sum of masses of belief committed to every subset of A by m [43]. This belief function, bel, represents the total belief that one commits to A without being also committed to $\overline{A}$. The belief function $bel : 2^{\Omega} \rightarrow [0,1]$ is defined so that:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B), \forall A \subseteq \Omega \tag{3}$$

$$bel(\emptyset) = 0. \tag{4}$$

The plausibility function $pl : 2^{\Omega} \rightarrow [0,1]$ quantifies the maximum amount of belief that could be given to a subset A of Ω. It is equal to the sum of the masses given to subsets B compatible with A:

$$pl(A) = \sum_{A \cap B \neq \emptyset} m(B), \forall A \subseteq \Omega \tag{5}$$

$$pl(\emptyset) = 0. \tag{6}$$

2.2. Special belief functions

In this subsection, we propose some belief functions used to express particular situations related generally to uncertainty. A *vacuous bba* is defined as follows [43]:

$$m(\Omega) = 1 \text{ and } m(A) = 0 \quad \forall A \neq \Omega\,. \tag{7}$$

Such function quantifies the state of total ignorance by having only Ω as a focal element.

A *categorical bba* is a normalized bba defined as follows:

$$m(A) = 1 \quad \forall A \subset \Omega \quad \text{and} \quad m(B) = 0 \quad \forall B \subseteq \Omega, B \neq A. \tag{8}$$

This function has a unique focal element different from the frame of discernment Ω.

A *certain bba* is a particular categorical bba such that its focal element is a singleton. A certain bba is defined as follows:

$$m(A) = 1 \text{ and } m(B) = 0 \quad \forall B \neq A \quad \text{and} \quad B \subseteq \Omega \quad \text{and} \quad |A| = 1 \tag{9}$$

where A is a singleton event of Ω. This function represents a state of total certainty on the focal element.

A *simple support function* (*ssf*) if it has at most one focal element different from the frame of discernment Ω. A simple support function is defined as follows [46]:

$$m(X) = \begin{cases} w & \text{if } X = \Omega \\ 1-w & \text{if } X = A \quad \forall A \subseteq \Omega \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where A is the focal element and $w \in [0,1]$. It presents a belief function induced by a piece of evidence supporting A (with $1 - w$) and leaving the remaining beliefs for Ω. This bba can also be noted A^w.

A *Bayesian bba* is a particular case of belief functions where all the focal elements are singletons. The corresponding bba is defined as follows:

$$m(A) > 0 \text{ only when } |A| = 1. \tag{11}$$

In this case, $bel = pl$ and they are considered as a probability distribution.

A *consonant bba* is a bba when all its focal elements ($A_1, A_2, \ldots, A_n$) are nested, that is $A_1 \subseteq A_2 \subseteq \ldots \subseteq A_n$. It is a special case of possibilities.

A *dogmatic belief function* is defined such that $m(\Omega) = 0$. Inversely a *non-dogmatic belief function* is defined such that $m(\Omega) > 0$ [46].

2.3. The discounting operation

Handling evidence given by experts requires to take into account the level of expertise of each information source. Indeed, reliability differs from one expert to another and a discounting method is imperative to update experts' beliefs based on weighting most heavily the opinions of the best experts and conversely for the less reliable ones.

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