# An analytical approach to evaluate point cloud registration error utilizing targets 

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#### Abstract

Point cloud registration is essential for processing terrestrial laser scanning (TLS) point cloud datasets. The registration precision directly influences and determines the practical usefulness of TLS surveys. However, in terms of target based registration, analytical point cloud registration error models employed by scanner manufactures are only suitable to evaluate target registration error, rather than point cloud registration error. This paper proposes an new analytical approach called the registration error ( $\boldsymbol{R E}$ ) model to directly evaluate point cloud registration error. We verify the proposed model by comparing $\boldsymbol{R E}$ and root mean square error (RMSE) for all points in three point clouds that are approximately equivalent.


## 1. Introduction

Terrestrial laser scanning (TLS) is used for a rapid collection of dense, three-dimensional (3D) spatial point cloud datasets of an entire object. Usually several scans are required with different stations to survey a relatively large and complex object completely due to occluded surfaces and scanner field of view limitations (Reshetyuk, 2009). To obtain the object's complete 3D model, the point cloud datasets must first be registered to a chosen coordinate system (Zhang, 2012).

Previous registration studies mainly include: (1) Matric representation for rotation transformation, such as Euler angle (Lichti and Skaloud, 2010; Zhang, 2008), unit quaternion (Lichti and Skaloud, 2010; Zhang, 2008; Yang, 2011), direction cosines (Lichti and Skaloud, 2010; Yang, 2011), dual quaternions (Walker and Shao, 1991), etc.; (2) Algorithms to compute 3-D rigid body transformation, such as singular value decomposition (Eggert et al., 1997; Arun et al., 1987), unit quaternion (Eggert et al., 1997; Faugeras and Hebert, 1986; Horn, 1987), dual quaternions (Walker and Shao, 1991; Eggert et al., 1997), orthonormal matric (Eggert et al., 1997; Horn et al., 1988), Lodrigues matric (Yao et al., 2007), etc.; (3) Iterative closest point method (ICP) (and variants), such as the feature correspondences (Salvi et al., 2007; Besl and Mckay, 1992; Tarel et al., 1998; Stamos and Leordeanu, 2003), registration strategy (Salvi et al., 2007; Chuang et al., 1998; Carmichael et al., 1999), correspondence search (Salvi et al., 2007; Chow et al.,

2004; Zinsser et al, 2003), robustness (Salvi et al., 2007; Chow et al., 2004; Zinsser et al, 2003), etc.; (4) Point cloud registration error models, such as error propagation for two scans (Wang, 2006), error propagation for multiple scans (Zhang, 2012; Wang, 2006; Sharp et al., 2004), directly geo-referenced TLS data precision (Lichti et al., 2005; Fan et al., 2015), the relationship between target precision and distribution relationships (Reshetyuk, 2009; Gordon and Lichti, 2004; Harvey, 2004; Bornaz et al., 2003), etc.

For target registration, point cloud registration error models and their statistics employed by scanner manufacturer software are based on how well the targets match. These approaches have been shown to be inadequate (Fan et al., 2015), since target registration error is not equal to the point cloud registration error. Although Fan et al. (2015) recommended a model to evaluate registration error based on how well the point clouds matched, However, the model was derived from simulations, which are not always consistent with actual outcomes since practical situations are often very complicated. Therefore, this paper derives the target based point cloud registration error model analytically, and verifies the model by evaluating real-world point cloud registration precision.

## 2. Estimation of registration parameters

We first introduce the common registration model to provide true

[^0]observation and transformation parameter values. We then consider true and approximate errors for these parameters, and derive the registration model error analytically using the estimation value and transformation parameter variances. Finally, we derive the analytical model to evaluate target based point cloud registration error.

### 2.1. Registration model

Target based registration of two scans is the most common registration approach and is most often performed using 3D rigid body transformation algorithm (Zhang, 2008; Eggert et al., 1997; Yao et al., 2007). The registration model can be expressed as point clouds in Scan $\boldsymbol{i}+1$ are transformed into Scan $\boldsymbol{i}$ using the true values of three translation parameters $\tilde{t_{x}}, \tilde{t_{y}}, \tilde{t_{z}}$ and three rotation parameters $\tilde{a}, \widetilde{b}, \widetilde{c}$ (Zhang, 2008; Yang, 2011),
$\widetilde{p}_{j}^{i}=\left[\begin{array}{c}\widetilde{x}_{j}^{i} \\ \widetilde{y}_{j}^{i} \\ \widetilde{z}_{j}^{i}\end{array}\right]=\widetilde{R}\left[\begin{array}{c}\widetilde{x}_{j}^{i+1} \\ \widetilde{y}_{j}^{i+1} \\ \widetilde{z}_{j}^{i+1}\end{array}\right]+\widetilde{T}=\widetilde{R} \widetilde{p}_{j}^{i+1}+\widetilde{T}$,
where $\widetilde{p}_{j}^{i}$ and $\widetilde{p}_{j}^{i+1}$ represent the coordinate true values of the same point in Scan $\boldsymbol{i}$ and Scan $\boldsymbol{i}+1$, respectively, i.e., $\left(\widetilde{x}_{j}^{i}, \widetilde{y}_{j}^{i}, \widetilde{z}_{j}^{i}\right)$ and $\left(\widetilde{x}_{j}^{i+1}, \widetilde{y}_{j}^{i+1}, \widetilde{z}_{j}^{i+1}\right) ; \widetilde{T}$ is a $3 \times 1$ translation vector,
$\widetilde{T}=\left[\begin{array}{c}\tilde{t}_{x} \\ \tilde{t}_{y} \\ \tilde{t}_{z}\end{array}\right]$,
and $\widetilde{R}$ is a $3 \times 3$ rotation matrix,
$\widetilde{R}$

$$
=\frac{1}{1+\widetilde{a}^{2}+\widetilde{b}^{2}+\widetilde{c}^{2}}\left[\begin{array}{ccc}
1+\widetilde{a}^{2}-\widetilde{b}^{2}-\widetilde{c}^{2} & 2(\widetilde{c}+\widetilde{a} \widetilde{b}) & 2(\widetilde{a} \widetilde{c}-\widetilde{b})  \tag{3}\\
2(\widetilde{a} \widetilde{b}-\widetilde{c}) & 1-\widetilde{a}^{2}+\widetilde{b}^{2}-\widetilde{c}^{2} & 2(\widetilde{a}+\widetilde{b} \widetilde{c}) \\
2(\widetilde{b}+\widetilde{a} \widetilde{c}) & 2(\tilde{b} \widetilde{c}-\widetilde{a}) & 1-\widetilde{a}^{2}-\widetilde{b}^{2}+\widetilde{c}^{2}
\end{array}\right]
$$

$\widetilde{R}^{T}=\widetilde{R}^{-1},|\widetilde{R}|=1$.
Let $\tilde{\eta}=\left[\widetilde{a}, \widetilde{b}, \widetilde{c}, \tilde{t_{x}}, \tilde{t_{y}}, \tilde{t_{z}}\right]^{T}$ be the vector of transformation parameters. To uniquely determine $\tilde{\eta}$ between Scan $\boldsymbol{i}$ and Scan $i+1$, we normally use three or more targets with known 3D coordinates (Reshetyuk, 2009; Bornaz et al., 2003), placed in the overlaps between the two point clouds. This paper assumes the number of targets is $k$ $(\geqslant 3)$, hence
$\left[\begin{array}{c}\widetilde{p}_{1}^{i} \\ \widetilde{p}_{2}^{i} \\ \vdots \\ \widetilde{p}_{k}^{i}\end{array}\right]=\left[\begin{array}{c}\widetilde{R} \widetilde{p}_{1}^{i+1} \\ \widetilde{R} \widetilde{p}_{2}^{i+1} \\ \vdots \\ \widetilde{R} \widetilde{p}_{k}^{i+1}\end{array}\right]+\left[\begin{array}{c}\widetilde{T} \\ \widetilde{T} \\ \vdots \\ \widetilde{T}\end{array}\right]$.

### 2.2. Error equation of target based registration model

Errors inevitably occur in TLS measurements (including instrumental errors, environmental errors, object related errors, target centroid errors, saturation errors, blooming errors, etc. (Reshetyuk, 2009)). If the observation values of $\widetilde{p}_{j}^{i}$ and $\widetilde{p}_{j}^{i+1}$ are $p_{j}^{i}$ and $p_{j}^{i+1}$, respectively, and approximate values of $\widetilde{R}, \widetilde{T}, \widetilde{\eta}$ are $R_{0}, T_{0}, \eta_{0}$ $\left(\eta_{0}=\left[a_{0}, b_{0}, c_{0}, t_{x 0}, t_{y 0}, t_{z 0}\right]^{T}\right.$ can be calculated by the method in Appendix $C$ ), then true errors of $p_{j}^{i}, p_{j}^{i+1}, R_{0}, T_{0}$, and $\eta_{0}$ are $\Delta_{p_{j}}, \Delta_{p_{j}^{i+1}}, \Delta_{R}, \Delta_{T}$, and $\Delta_{\eta}$ respectively, where
$\widetilde{p}_{j}^{i}=p_{j}^{i}+\Delta_{p_{j}}, \widetilde{p}_{j}^{i+1}=p_{j}^{i+1}+\Delta_{p_{j}^{i+1}}$,
$\widetilde{R}=R_{0}+\Delta_{R_{0}}, \widetilde{T}=T_{0}+\Delta_{T_{0}}$,
and
$\tilde{\eta}=\eta_{0}+\Delta_{\eta_{0}}$.
Hence, from Eq. (5),
$v_{j}=\Delta_{R} \cdot p_{j}^{i+1}+\Delta_{T}-l_{j}$,
where $l_{j}=p_{j}^{i}-R_{0} p_{j}^{i+1}-T_{0}, j \in\{1,2, \ldots, k\}, v_{j}=-\left(R_{0} \Delta_{p_{j}^{i+1}}+\Delta_{R} \Delta_{p_{j}^{i+1}}\right) \quad$ is residual error.

Using the linearization theorem (Qiu et al., 2008),
$\left\{\begin{array}{l}\Delta_{R} \approx d R=\frac{\partial R}{\partial a} d a+\frac{\partial R}{\partial b} d b+\frac{\partial R}{\partial c} d c \\ \Delta_{T} \approx d T=\left[d t_{x}, d t_{y}, d t_{z}\right]^{T} \\ \Delta_{\eta} \approx d \eta=\left[d a, d b, d c, d t_{x}, d t_{y}, d t_{z}\right]^{T}\end{array}\right.$,
where $d R, d T, d \eta$ are the approximate values for $\Delta_{R}, \Delta_{T}, \Delta_{\eta}$, respectively.

We can construct the error equations of the target based registration model from Eqs. (6) and (7),
$V \approx B \cdot d \eta-l$,
where $V$ and $l$ are $3 k \times 1$ matrices, $B$ is a $3 k \times 6$ matrix,
$V=\left[\begin{array}{c}v_{1} \\ v_{2} \\ \vdots \\ v_{k}\end{array}\right], B=\left[\begin{array}{c}B_{1} \\ B_{2} \\ \vdots \\ B_{k}\end{array}\right], l=\left[\begin{array}{c}l_{1} \\ l_{2} \\ \vdots \\ l_{k}\end{array}\right], v_{j} \approx d R \cdot p_{j}^{i+1}+d T-l_{j}=B_{j} \cdot d \eta-l_{j}$,
$R_{0}$

$$
=\frac{1}{1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}}\left[\begin{array}{ccc}
1+a_{0}^{2}-b_{0}^{2}-c_{0}^{2} & 2\left(c_{0}+a_{0} b_{0}\right) & 2\left(a_{0} c_{0}-b_{0}\right)  \tag{10}\\
2\left(a_{0} b_{0}-c_{0}\right) & 1-a_{0}^{2}+b_{0}^{2}-c_{0}^{2} & 2\left(a_{0}+b_{0} c_{0}\right) \\
2\left(b_{0}+a_{0} c_{0}\right) & 2\left(b_{0} c_{0}-a_{0}\right) & 1-a_{0}^{2}-b_{0}^{2}+c_{0}^{2}
\end{array}\right],
$$

$T_{0}=\left[\begin{array}{c}t_{x 0} \\ t_{y 0} \\ t_{z 0}\end{array}\right], p_{j}^{i+1}=\left[\begin{array}{c}x_{j}^{i+1} \\ y_{j}^{i+1} \\ z_{j}^{i+1}\end{array}\right]$,
$B_{j}=\left[\begin{array}{llll}\frac{\partial R}{\partial a} p_{j}^{i+1} & \frac{\partial R}{\partial b} p_{j}^{i+1} & \frac{\partial R}{\partial c} p_{j}^{i+1} & E_{3 \times 3}\end{array}\right], E_{3 \times 3}=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$,
$\left\{\begin{array}{l}\frac{\partial R}{\partial a}=\left[\begin{array}{ccc}\frac{4 a_{0}\left(b_{0}^{2}+c_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2 b_{0}\left(1-a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)-4 a_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2 c_{0}\left(1-a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)+4 a_{0} b_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} \\ \frac{2 b_{0}\left(1-a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)+4 a_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{-4 a_{0}\left(1+b_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2\left(1-a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)-4 a_{0} b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} \\ \frac{2 c_{0}\left(1-a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)-4 a_{0} b_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{-2\left(1-a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)-4 a_{0} b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{-4 a_{0}\left(1+c_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}}\end{array}\right] \\ \frac{\partial R}{\partial b}=\left[\begin{array}{ccc}\frac{-4 b_{0}\left(1+a_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2 a_{0}\left(1+a_{0}^{2}-b_{0}^{2}+c_{0}^{2}\right)-4 b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{-2\left(1+a_{0}^{2}-b_{0}^{2}+c_{0}^{2}\right)-4 a_{0} b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} \\ \frac{2 a_{0}\left(1+a_{0}^{2}-b_{0}^{2}+c_{0}^{2}\right)+4 b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{4 b_{0}\left(a_{0}^{2}+c_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2 c_{0}\left(1+a_{0}^{2}-b_{0}^{2}+c_{0}^{2}\right)-4 a_{0} b_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} \\ \frac{2\left(1+a_{0}^{2}-b_{0}^{2}+c_{0}^{2}\right)-4 a_{0} b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2 c_{0}\left(1+a_{0}^{2}-b_{0}^{2}+c_{0}^{2}\right)+4 a_{0} b_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{-4 b_{0}\left(1+c_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}}\end{array}\right] \\ \frac{-4 c_{0}\left(1+a_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} \\ \frac{2\left(1+a_{0}^{2}+b_{0}^{2}-c_{0}^{2}\right)-4 a_{0} b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}}\end{array} \frac{2 a_{0}\left(1+a_{0}^{2}+b_{0}^{2}-c_{0}^{2}\right)+4 b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}}\right]\left[\begin{array}{lll}\frac{-2\left(1+a_{0}^{2}+b_{0}^{2}-c_{0}^{2}\right)-4 a_{0} b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{-4 c_{0}\left(1+b_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2 b_{0}\left(1+a_{0}^{2}+b_{0}^{2}-c_{0}^{2}\right)-4 a_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} \\ \frac{2 a_{0}\left(1+a_{0}^{2}+b_{0}^{2}-c_{0}^{2}\right)-4 b_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{2 b_{0}\left(1+a_{0}^{2}+b_{0}^{2}-c_{0}^{2}\right)+4 a_{0} c_{0}}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}} & \frac{4 c_{0}\left(a_{0}^{2}+b_{0}^{2}\right)}{\left(1+a_{0}^{2}+b_{0}^{2}+c_{0}^{2}\right)^{2}}\end{array}\right]$.

Assuming the weight matrix of $l$ is $P$, by using the principle of indirect adjustment (Qiu et al., 2008) and $V^{T} P V=\min$, we can obtain estimated $\hat{\eta}, \widehat{R}, \widehat{T}$ for transformation parameters $\widetilde{\eta}, \widetilde{R}, \widetilde{T}$ as

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