ISPRS Journal of Photogrammetry and Remote Sensing 141 (2018) 46–58

Contents lists available at ScienceDirect

ISPRS Journal of Photogrammetry and Remote Sensing

journal homepage: www.elsevier.com/locate/isprsjprs

ℓ_0-based sparse hyperspectral unmixing using spectral information and a multi-objectives formulation ☆

Xia Xu, Zhenwei Shi *, Bin Pan

Image Processing Center, School of Astronautics, Beihang University, Beijing, PR China
State Key Laboratory of Virtual Reality Technology and Systems, Beihang University, Beijing, PR China
Beijing Key Laboratory of Digital Media, Beihang University, Beijing, PR China

ARTICLE INFO

Article history:
Received 4 July 2017
Received in revised form 29 March 2018
Accepted 19 April 2018

Keywords:
Hyperspectral image
Sparse unmixing
Multi-objective
Non-convex optimization

ABSTRACT

Sparse unmixing aims at recovering pure materials from hyperpspectral images and estimating their abundance fractions. Sparse unmixing is actually ℓ_0 problem which is NP-h ard, and a relaxation is often used. In this paper, we attempt to deal with ℓ_0 problem directly via a multi-objective based method, which is a non-convex manner. The characteristics of hyperspectral images are integrated into the proposed method, which leads to a new spectra and multi-objective based sparse unmixing method (SMoSU). In order to solve the ℓ_0 norm optimization problem, the spectral library is encoded in a binary vector, and a bit-wise flipping strategy is used to generate new individuals in the evolution process. However, a multi-objective method usually produces a number of non-dominated solutions, while sparse unmixing requires a single solution. How to make the final decision for sparse unmixing is challenging. To handle this problem, we integrate the spectral characteristic of hyperspectral images into SMoSU. By considering the spectral correlation in hyperspectral data, we improve the Tchebycheff decomposition function in SMoSU via a new regularization item. This regularization item is able to enforce the individual divergence in the evolution process of SMoSU. In this way, the diversity and convergence of population is further balanced, which is beneficial to the concentration of individuals. In the experiments part, three synthetic datasets and one real-world data are used to analyse the effectiveness of SMoSU, and several state-of-art sparse unmixing algorithms are compared.

1. Introduction

Benefit from the development of remote sensing technique, imagery spectral resolution has been improved significantly and hyperspectral observation capability is formed. Hyperspectral images usually contain hundreds of spectrum bands, covering visible to thermal-infrared regions (Ma et al., 2014; Zhong et al., 2018). The abundant spectral information of hyperspectral image contributes to many practical applications, such as environmental monitoring and geological exploration (Van Ruitenbeek et al., 2006; Pan et al., 2017a). However, the spatial resolution of hyperspectral images is usually low and different features are always homogeneously mixed (Willett et al., 2014; Wang et al., 2017). Thus a single pixel always contains more than one land cover types, resulting in mixed pixels. The complex mixing of different features brings great challenge to hyperpsectral image processing (Pan et al., 2016; Wang et al., 2016; Zhou and Wei, 2016; Pan et al., 2017b). Unmixing aims at recovering pure materials spectra (*endmembers*) of a hyperspectral image, as well as their corresponding fractions (*abundances*). The abundance values are proportions representing the percentage of endmembers in a pixel region (Keshava and Mustard, 2002). Accordingly, hyperspectral unmixing is generally processed under two steps: (i) identifying the endmembers, (ii) quantifying the abundance fractions (Bioucas-Dias et al., 2012; Zhong et al., 2016).

Sparse regression based unmixing is a hot topic in recent years, which does not need to assume pure materials in hyperspectral images. Due to the simplicity and flexibility, linear mixing model (LMM) is the most widely used (Bioucas-Dias et al., 2012; Heylen et al., 2014; Zhu et al., 2014b). LMM characterizes the mixture without considering the effects of multiple scattering and intimate mixture. In sparse unmixing, the mixed image data is represented by pure signatures from a spectral library which is known in advance

☆ The work was supported by the National Key R&D Program of China under the Grant 2017YFC1405605, the National Natural Science Foundation of China under the Grant 61671037, and the Excellence Foundation of BUAA for PhD Students under the Grants 2017057.

* Corresponding author at: Image Processing Center, School of Astronautics, Beihang University, Beijing, PR China.

E-mail addresses: xuxia@buaa.edu.cn (X. Xu), shizhenwei@buaa.edu.cn (Z. Shi), panbin@buaa.edu.cn (B. Pan).

https://doi.org/10.1016/j.isprsjprs.2018.04.008
0924-2716/

(Iordache et al., 2011). In this way the estimation of abundances is no longer dependent on the presence of pure pixels. Sparse unmixing aims at determining an optimal subset of the pure materials, which may be a very small proportion relative to the library. Mathematically speaking, it is a ℓ_0 norm-based combinational problem, which is non-convex and NP-hard. The most commonly used approach to solve this problem is approximating it to a convex ℓ_1 norm regularized optimization problem (Bioucas-Dias and Figueiredo, 2010; Iordache et al., 2014a). To further approximate the ℓ_0 norm, $\ell_p(0 < p < 1)$ norm based methods are developed (Qian et al., 2011; Zhu et al., 2014a). In order to obtain a better unmixing performance, quite a few studies consider exploring the characteristics of hyperspectral image and the spectral library, such as the contextual information among pixels (Feng et al., 2014; Feng et al., 2016), the signature differences between hyperspectral image and spectral library (Shi and Wang, 2014; Zare and Ho, 2014). However, ℓ_1 or ℓ_p norm is only a relaxation of the original ℓ_0 problem. Using such a relaxation may lead to errors in the unmixing results. Another type of works try to deal with the ℓ_0 norm problem directly using greedy algorithms. In Shi et al. (2014), a subspace matching pursuit sparse unmixing method was proposed. Considering the high correlation of spectral library, the greedy selection is conducted based on the whole hyperspectral image data. In (Tang et al., 2014), the forward greedy step and the backward greedy step was combined to provide a more stable selection and less probabilities of local optima. Although greedy-based methods need no approximation, there are many sensitive parameters which have to adjust manually. Moreover, these methods usually encounter problems of endmember missing and redundancy (Shi et al., 2014).

Recently, multi-objective optimization (MO) has been proposed to solve ℓ_0 norm-based problems directly without any smoothing (Deb and Jain, 2014; Jain and Deb, 2014; Ma et al., 2015; Sun et al., 2016; Zhang and Tao, 2017; Ma et al., 2018). The major target of MO is to find a non-dominated solution set which provides a good trade-off for objective functions (Miettinen, 1999; Deb and Kalyanmoy, 2001). MO has made great progress in combinatorial optimization (Li et al., 2014Xue et al., 2016). Researchers have verified that MO can provide good solutions for many NP-hard problems, where it can recover the best-so-far guaranteed approximate solution within limited iterations. In Yu et al. (2013), an isolation-based MO framework was proposed, which could achieve the best-achievable result on minimum k-set cover problem with an H_k-approximation ratio. In the next few years, Qian et al. further explored the evolutionary problem and provided many theoretical supports on some NP-hard problems. In Qian et al. (2015a), Pareto optimization, penalty function method and greedy algorithms were compared theoretically on the minimum cost coverage problem. Pareto optimization was proved more efficient than penalty function method. It was also found to be positive on a special case of the problem, when compared with greedy algorithm. In Qian et al. (2015b), Pareto optimization based subset selection (POSS) was found that it was able to achieve the best-so-far approximation guarantee obtained by greedy algorithms on sparse regression. Later, POSS is sped up through paralleling (Qian et al., 2016).

For sparse unmixing, the balance of reconstruction error and endmember sparsity is precisely in line with the goal of MO. However, different from single-objective optimization, a number of conflicting objective functions need to be optimized simultaneously in MO (Deb and Kalyanmoy, 2001). In other words, it is difficult to select a specific solution that is optimal to the two objective functions. Therefore, MO is expected to determine a set of non-dominated solutions as close as the Pareto-optimal front to give a trade-off among objectives (Zhou et al., 2011). In practical applications such as sparse unmixing, there must be a specific strategy to determine the unique solution.

There are few works that try to handle the sparse unmixing problem by MO. In Xu and Shi (2017), sparse unmixing was transformed into a bi-objective optimization problem using POSS and non-dominated sorting genetic algorithm-II (NSGA-II). This method was a semi-automatic manner which requires selecting an individual from the final solution set manually. In Gong et al. (2017), considering the large scale of spectral library and heavy computing load caused by high dimensional problem, the spectral library was grouped and a cooperation strategy was designed among the groups. This algorithm optimized the abundance matrices and used the knee point in the Pareto front as the final solution. However, although these methods have presented good performance, how to determine the final solution from the obtained non-dominated front is still challenging for sparse unmixing. If suboptimal point is picked as the final solution, the endmember missing or redundancy is likely to happen. In this case, the unmixing accuracy is suffered significantly. Furthermore, endmember sparsity in most existing methods relies heavily on the parameter settings of the mutation and crossover operator, which may result in an inadequate sparsity.

In this paper, a new multi-objective optimization based sparse unmixing method is proposed, which takes full advantage of the spectral characteristic in hyperspectral images. We term the proposed work as integrating spectra and multi-objective for ℓ_0 sparse unmixing (SMoSU). SMoSU is developed under the framework of the multi-objective evolutionary algorithm based on decomposition (MOEA/D) (Zhang and Li, 2007). MOEA/D could provide a non-Pareto criterion, which leads to a fast evolutionary speed and low computational complexity. In SMoSU, sparse unmixing is transformed to a bi-objective discrete optimization problem, where reconstruction error and endmember sparsity error are taken as the two conflicting objectives. Inspired by the binary coding of minimum k-set cover problem in Yu et al. (2013), we encode the spectral library in a binary vector and use a bit-wise flipping strategy for individual generation. Each bit of the binary representation indicates the corresponding spectrum. The probability-based bit-wise flipping is verified to be effective in global search and does not need to adjust many parameters (Yu et al., 2013; Qian et al., 2015b). However, as is discussed above, a set of non-dominated solutions are generated by MOEA/D, which is hard to select a final solution. In this paper, we improve the Tchebycheff decomposition approach in MOEA/D by integrating the spectral characteristic of hyperspectral data. The improved decomposition function takes spectral correlations among individuals into account. A regularization item that enforces the individual divergence is included in SMoSU. By this means the diversity and convergence of population are further balanced, which is beneficial to the concentration of individuals. The major contributions of SMoSU can be summarized as follows:

- We introduce a new multi-objective based sparse unmixing method which could solve the ℓ_0 non-convex optimization without any relaxation. The reconstruction error and endmember sparsity error are considered as two objectives in SMoSU.
- We use a binary code strategy for spectral signatures in the library and proposed a bit-wise flipping approach for individual generation based on the framework of MOEA/D. In this case, the optimal subset selection of spectral library is transformed to finding an optimal binary vector.
- To overcome the difficulty of selecting the final solution, we integrate the spectral characteristic of hyperspectral images into the multi-objective framework. The Tchebycheff decomposition approach is regularized by the spectral correlations among different individuals, so as to further balance the population divergence and convergence.

The rest of this paper is organized as follows. Section 2 introduces the background about sparse unmixing and MOEA/D.