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Dealing with periodic disturbances in controls of mechanical systems☆

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Abstract

Periodic disturbances are common in control of mechanical systems. Such disturbances may be due to rotational elements such as motors and vibratory elements. When the period of a periodic disturbance is fixed and known in advance, repetitive control can be used for attenuating their effect. The most popular repetitive controller is based on the internal model principle. When the period is not fixed and unknown, adaptation capability must be introduced. This paper presents some fundamental issues and new challenges in the design of controllers to deal with periodic disturbances along with applications to mechanical systems.

Keywords: Repetitive control; Learning control; Mechanical systems; Periodic disturbances

1. Introduction

Reference inputs and disturbances are often periodic in mechanical control applications. Periodic disturbances may be due to rotational components such as motors, and in such cases the period may be assumed to be known or measurable. On the other hand, if the periodic disturbance comes from vibrations external to the servo control loop, the period may not be known in advance. Repetitive control was originally formulated by Inoue, Nakano, and Iwai (1981) to deal with repetitive disturbances with a known period, and was developed by many researchers (Hara, Yamamoto, Omata, & Nakano, 1988; Hu & Tomizuka, 1993; Tomizuka, Tsao, & Chew, 1989; Yamada, Riadh, & Funahashi, 1999 among others). Successful applications of repetitive control include machining (Tsao & Tomizuka, 1994) and computer hard disk drives (Chew & Tomizuka, 1990). Early work on repetitive control was based on the key assumption that the period of repetitive disturbance is precisely known. While this assumption holds in many applications, more recent research efforts have been directed at dealing with cases where the period is not known in advance or is time varying (Hu, 1992; Tsao & Nemani, 1992; Tsao, Qian, & Nemani, 2000).

Repetitive control attempts to compensate for all repetitive frequency components, the fundamental frequency component as well as all higher order harmonics. In many cases, this is not necessary. For example, if a control system is perturbed by a single sinusoidal disturbance, it suffices if the compensator is designed for the single frequency component. Peak filters popular in hard disk drive (HDD) controls represent an example (Kim, Kang, & Tomizuka, 2005) of this methodology. Such approaches may be easier to extend to adaptive cases including cases for repetitive signals with unknown periods (Landau, Constantinescu, & Rey, 2005).

Closely related to repetitive control is iterative learning control. Iterative learning control was motivated by robots that must perform a repetitive operation over a finite time interval (Uchiyama, 1978). Early work on the subject most frequently cited is the betterment approach by Arimoto, Kawamura, and Miyazaki (1984), but there have been other independent developments of similar ideas at about the same time as Arimoto (Longman, 2000). Iterative learning control and repetitive control have formed a substantial research community.

The objective of this paper is to provide the fundamental design and implementation issues of repetitive control as related to the original discrete time repetitive controller (Tomizuka et al., 1989), and to introduce selected recent research activities on compensations for periodic disturbances. Application examples are drawn from mechanical systems.

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2. Basics of repetitive control

Consider a discrete time system described by

$$\begin{aligned} A(z^{-1})y(k) &= z^{-d}B(z^{-1})[u(k)+w(k)] \\ A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \cdots + b_m z^{-m} \end{aligned} \tag{1}$$

where u, y and d are, respectively, the input, output and disturbance, z^{-1} represents one time step delay, and d is the pure delay steps. Note that the input–output transfer function is

$$P(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \tag{2}$$

Assume that the system is asymptotically stable, i.e. the poles of the transfer function are all inside the unit circle. $B(z^{-1})$ is written as

$$\begin{aligned} B(z^{-1}) &= B^{C}(z^{-1})B^{u}(z^{-1}) \\ B^{c}(z^{-1}) &= b_0 + b_1^{c} z^{-1} + \cdots + b_{mc}^{c} z^{-mc} \\ B^{u}(z^{-1}) &= 1 + b_1^{u} z^{-1} + \cdots + b_{mu}^{u} z^{-mu} \end{aligned} \tag{3}$$

where $B^{c}(z^{-1})$ and $B^{u}(z^{-1})$ contain, respectively, cancelable zeros and uncancelable zeros. Uncancelable zeros include all unstable zeros, i.e. those on and outside the unit circle.

The disturbance $w(k)$ is repetitive with period N, i.e.

$$(1 - z^{-N})w(k) = 0 \tag{4}$$

The control objective is to achieve asymptotic regulation of the output error, i.e.

$$\lim_{k\to\infty} e(k) = 0 \tag{5}$$

where $e(k) = y_{\mathrm{d}}(k)\ \ y(k)$ and y_{d} is the desired output. $y_{\mathrm{d}}(k)$ is constant or periodic with period N.

For repetitive (periodic) desired outputs and disturbances with period N, asymptotic regulation may be achieved by the controller in several different ways. Among alternatives, repetitive control based on the internal model principle (Francis & Wonham, 1975) is most popular, and it has been studied most extensively. Based on the internal model principle, the feedback controller for (1) needs the internal model of repetitive signals for asymptotic regulation of the error. Such a controller may be represented by

$$U(z) = \frac{k_r z^{-N+d} A(z^{-1}) B^{u}(z)}{(1 - z^{-N}) B^{c}(z^{-1}) b} E(z), \quad b > \max_{\omega \varepsilon [0,\pi]} |B^{u}(e^{j\omega})|^2 \tag{6}$$

where $1/(1 - z^{-N})$ is the internal model of repetitive signals. This is easily understood in view of (4).

$B^{u}(z)$ in Eq. (6) is not realizable, but $B^{u}(z)z^{-mu}$ is realizable. Noting this, a realizable implementation of the repetitive controller becomes as depicted in Fig. 1.

The following result has been obtained by Tomizuka et al. (1989).

2.1. Theorem

The feedback system (repetitive control system) consisting of (1) and (6) is asymptotically stable for $0 < k_r < 2$, and asymptotic regulation of the output error is achieved for repetitive disturbances and reference inputs with period N.

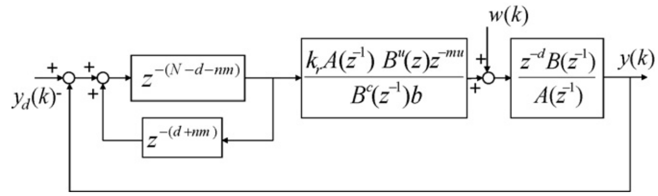

Fig. 1. Repetitive control system.

We describe below several aspects of the repetitive control system, which are important, in particular, from the viewpoint of applications.

2.2. Stability robustness

The stability condition in the theorem suggests that the repetitive control system may be robust for parameter variation by selecting the gain, k_r, small. It was, however, noticed at an early stage of theory development that the stability of repetitive control systems with the exact internal model of repetitive signals is not robust in the presence of unmodelled (parasitic) dynamics. This problem arises due to the nature of the internal model, i.e. the characteristic roots of the internal model, $1/(1 - z^{-N})$, are all on the unit circle, which is the stability boundary. Another way to interpret this point is the inherent high gain nature of the repetitive controller; the frequency response of the internal model goes to infinity at every repetitive frequency. This problem may be overcome by introducing a low pass filter in the internal model. The repetitive controller with a modified internal model is

$$U(z) = \frac{k_r q(z, z^{-1}) z^{-N+d+mu} A(z^{-1})}{(1 - q(z, z^{-1}) z^{-N}) B^{c}(z^{-1}) b},$$

$$b > \max_{\omega \varepsilon [0,\pi]} |B^{u}(e^{j\omega})|^2, \quad q(z, z^{-1}) = \frac{\alpha_1 z^{-1} + \alpha_0 + \alpha_1 z}{\alpha_0 + 2\alpha_1} \tag{7}$$

Notice that $q(z,z^{-1})$ is a low pass filter with zero phase characteristics (Tomizuka, 1987). Notice also that $q(z,z^{-1})$ is not realizable, but (7) is realizable as long as $N - d - mu > 0$. The order of q-filter may be increased by introducing higher order terms of z and z^{-1}.

If we define the system modeling uncertainty by

$$r(e^{-j\omega}) = \frac{P_0(e^{-j\omega}) - P(e^{-j\omega})}{P_0(e^{-j\omega})} \tag{8}$$

where $P_0(e^{-jw})$ represents the nominal dynamics of the system and $P(e^{-jw})$ the actual dynamics, q-filter must be selected to satisfy the following condition for robust stability (Tsao & Tomizuka, 1994),

$$\frac{1}{|r(e^{-jw})|} \geq |q(e^{jw}, e^{-jw})| \tag{9}$$

The q-filter offers robust stability. On the other hand, the modified internal model is no longer a generator of periodic signals. Thus, asymptotic regulation does not take place, but for