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Solving bundle block adjustment by generalized anisotropic Procrustes analysis

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ABSTRACT

The paper presents a new analytical tool to solve the classical photogrammetric bundle block adjustment. The analytical model is based on the generalized extension of the anisotropic row-scaling Procrustes analysis, that has been recently proposed by the same authors to solve the image exterior orientation problem. The main advantage of the method is given by the fact that the problem solution does not require any approximate value of the unknown parameters, nor any linearization procedure. Moreover, the algorithm is exceedingly simple to describe and easy to implement. Empirical results indicate that a zero-information initialization of the iterative relaxation procedure leads almost always to the correct final least squares solution. Experiments confirm the accuracy of the proposed method, when compared to the results obtained by applying a classical photogrammetric bundle block adjustment.

1. Introduction

Bundle block adjustment is the most classical analytical problem in photogrammetry. Great effort was given to the problem solution since the middle of the last century (e.g., Baetsle, 1956; Brown, 1958; Ackermann, 1962; Cunietti, 1968). An exhaustive synthesis of the analytical developments carried out in this fundamental field of photogrammetry can be found, for instance, in Triggs et al. (2000). The standard formulation requires the linearization of the collinearity equations and the satisfaction of the least squares principle for the equation residuals. Some additional unknown terms can be considered for each equation in order to calibrate the camera for some systematic error terms or for simultaneously estimating the image interior orientation parameters. Robust least squares solutions have been also proposed in the literature in order to reduce the influence of outliers (e.g., Zhang et al., 2006).

According to Triggs et al. (2000), the most significant bundle block adjustment paradigmatic enhancements in chronological sequence, are:

1. recursive partitioning by Gyer (1967) and Brown (1968) that led to the modern sparse matrix techniques;
2. S transformations and criterion matrices by Baarda (1973), that allowed the correct estimate of the network degrees of freedom and the uncertainty modeling in the adjustment process;
3. photogrammetric precision and reliability over-parametrization and model choice by Gruen (1980) and Foerstner (1985), that opened the way to modern robust statistics and model selection in photogrammetry;
4. "geometrically constrained multiphoto and globally enforced least squares matching" by Gruen and Baltsavias (1986), that introduced the so called image-based matching technique procedures.

In spite of these fundamental steps in the methodological development of bundle adjustment, the common underlying scheme, based on the least-squares solution of a large non-linear system of equations, has been the same since its origins. In this paper we propose instead a new analytical bundle block solution method rooted in the framework of orthogonal Procrustes analysis, in particular focusing on its *generalized anisotropic* variant. The main advantage of the method is that – upon convergence – it furnishes a least squares solution without any linearization of the original equations, and without any approximate value of the unknown parameters and of the tie-points 3-D coordinates.

Recently, the authors of this paper applied the anisotropic row-scaling Procrustes analysis technique to successfully solve the exterior orientation problem of one image (Garro et al., 2012). The process is carried out by an iterative relaxation of the

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unknown translation, rotation and anisotropic scaling of each image point.

This paper, in particular, extends the same relaxation procedure by considering also the presence of unknown tie-points imaged to some or all exposures. Their role is to constraint the different images one to each other in order to iteratively update their approximate exterior orientation parameters until a global convergence of the entire block.

The method introduced in this paper represents a further extension of the generalized (isotropic) Procrustes analysis, already applied for the least squares registration of photogrammetric 3-D models (e.g. Crosilla and Beinat, 2002), where just one isotropic scale factor is required for all the points of the same model.

The new method falls in the wider structure-from-motion family. Its closest neighbors are the *iterative factorization* methods and the *global motion-first* methods.

Iterative factorization methods (Sturm and Triggs, 1996; Heyden, 1997; Oliensis, 1999; Oliensis and Hartley, 2007) yield a projective model from multiple images by a two step iteration (a block relaxation, in fact), where in one step a measurement matrix, containing image points coordinates, is factorized with SVD, and in the subsequent step the depths of the points are computed, assuming all the other parameters fixed. This bears some resemblance of the scheme described in this paper, which however, deals with calibrated cameras (i.e., known interior orientation) and outputs a Euclidean model[1] instead of a projective one. Moreover, unlike these algorithms, our method does not require all points to be visible in all views.

The issue of visibility in matrix factorization methods can be side-stepped by matrix completion techniques, exploiting the low rank of the measurement matrices (Brand, 2002; Kennedy et al., 2013; Hartley and Schaffalitzky, 2003), or by providing additional information. Indeed, Kaucic et al. (2001) and Rother and Carlsson (2002), describe algorithms based on SVD for the projective modeling from multiple perspective views, based on having four points on a reference plane visible in all views. Unlike iterative factorization ones, these algorithms does not require all points to be visible in all views and are also direct. If three orthogonal vanishing points are specified in addition, the model can be upgraded to Euclidean. Hartley and Schaffalitzky (2003) scheme, in particular, can be contrasted with ours, since its iteration can be interpreted as linearly solving alternately for camera matrices and 3D points until their product converges to the measurement matrix.

Projective methods, though, respond to a practical situation (i.e., unknown interior orientation), which is different from the one addressed in this paper. Moreover, we do not put any constraint on the input (like having four points on a reference plane visible in all views).

Global motion-first methods share a common scheme: they start from known interior orientation, compute epipolar or trifocal geometry which results in relative rotations and relative translations (up to a scale). Then solve the *rotation registration* or *rotation averaging* (Hartley et al., 2013) problem that gives the rotational component of the cameras orientation; this problem, if one ignores outliers, can be solved directly by eigen-decomposition of a matrix (Martinec and Pajdla, 2007; Arie-Nachimson et al., 2012). Finally, camera location is solved (a.k.a. *translation registration*) by a variety of direct/iterative methods, including solving a linear system of equations (Kraus, 1997, Section 4.1; Arie-Nachimson et al., 2012; Jiang et al., 2013), eigen-decomposition (Brand et al., 2004), linear programming (Moulon et al., 2013), Second Order Cone programming (Kahl and Hartley, 2008; Martinec and Pajdla, 2007), non-linear least squares (Wilson and Snavely, 2014).

Since some of these methods are direct and the scheme proposed in this paper is iterative, they can be considered superior from this point of view. However, they minimize algebraic residuals, which in certain cases are based only on the orientations (e.g. Brand et al., 2004), ignoring the 3D points until the final intersection. On the other hand, our method minimizes a *geometric residual*, similarly to photogrammetric bundle block adjustment. As a matter of fact, the experiments reported show that the method introduced in this paper achieves RMS error values (wrt. ground control points) practically equal to those obtained by photogrammetric bundle block adjustment.

2. Procrustes analysis and photogrammetry

Let us start this section by summarily presenting the main characteristics of the generalized (isotropic) Procrustes analysis in Photogrammetry and laser scanning applications. Afterwards, the anisotropic row-scaling Procrustes analysis will be presented, and its capabilities to successfully solve the exterior orientation of one image and the bundle block adjustment problem will be emphasized.

2.1. Registration of multiple 3-D models

As well known, photogrammetric relative orientation and laser scanning can provide numerical 3-D models of real objects by sampling the positions of a set of representative surface points. Depending on the extension and on the shape complexity of the geometric entity to be surveyed, its complete acquisition often leads to the creation of a set of partial and independent 3-D models. These parts must be joined together to reconstruct the complete object model into a unique frame by matching common points or features, or by directly aligning portions of corresponding surfaces.

The registration of multiple 3-D models or *n-view registration* problem requires to simultaneously transform into a unique mean coordinate system a set of $m \geqslant 2$ models, each composed of n points coordinates in $\mathbb{R}^3$ defined in a different reference frame.

If these models are expressed by m $n \times 3$ matrices $A_1, \ldots, A_m$, the problem is equivalent to:

$$\min \sum_{i<j}^{m} \left\| (\lambda_i A_i R_i + \mathbf{1}\mathbf{t}_i^T) - (\lambda_j A_j R_j + \mathbf{1}\mathbf{t}_j^T) \right\|_F^2 \tag{1}$$

where $\mathbf{1}$ is the all-ones vector and $(\lambda_i, R_i, \mathbf{t}_i)$ are the parameters of a similarity (a.k.a. Helmert) transformation. This is a generalized (isotropic) Procrustes analysis (GPA) model (Gower, 1975), whose solution allows to directly register all the 3-D models into a unique mean reference frame, minimizing a geometric error.

The analogy with the photogrammetric independent models block adjustment is evident:

- the number of the A_i matrices is equal to the number m of the models composing the block adjustment. The matrices contain the coordinates of the available points for each model;
- all matrices A_i have the same dimension, equal to the global number n of the block adjustment observations by the coordinate space dimension (usually 3);
- in the case of missing data, the generic matrix A_i has specified components only for the points belonging to the i-th model, the other ones being unspecified.

Fig. 1 explains these positions.

[1] A projective/Euclidean model differs from the true one by a projectivity/similarity transformation.