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Exponential convergence of a nonlinear attitude estimator ☆

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ABSTRACT

A nonlinear attitude estimator is presented. The estimator uses a minimum of two vector measurements and a rate-gyro to estimate the attitude and the rate-gyro bias. The estimator can be implemented using a low-cost inertial measurement unit such as those typically used on inexpensive aerial vehicles. Estimator design is based on an alternate attitude error function that yields an estimator with faster convergence properties than similar estimators previously considered in the literature. A proof of exponential stability about the desired equilibrium point is provided. Simulation and experimental results demonstrate the desirable properties of the proposed estimator.

1. Introduction

Attitude estimation is the process of determining the orientation, or attitude, of one reference frame relative to another reference frame. It is vital to the navigation, guidance, and control of autonomous vehicles, as well as to the operation of many consumer electronics. The closed-loop control of miniature aerial vehicles, for example, requires accurate attitude estimates. In fact, poor attitude estimates may result in poor performance and even instability of these vehicles. The development of reliable and robust attitude estimation algorithms are therefore of great practical interest.

Cost is also a concern in the development of attitude determination and control systems. The issue of cost is particularly evident in the consumer electronics marketplace where demand for inertial sensing hardware has lead to the development of low-cost inertial measurement units (IMUs). These IMUs are generally equipped with an accelerometer, a magnetometer, and a rate-gyro. It is possible to obtain an attitude estimate using only accelerometer and magnetometer measurements using the TRIAD algorithm, QUEST, or others (Markley & Crassidis, 2014). Unfortunately, the accelerometers and magnetometers of low-cost IMUs are corrupted by high levels of noise and bias (Mahony, Hamel, & Pflimlin, 2008). Typically, filters that fuse accelerometer and magnetometer data with rate-gyro measurements are used in practice. One such method, the multiplicative extended Kalman filter (MEKF), is a popular attitude estimation technique and has been widely used for spacecraft attitude estimation (Markley, 2003). However, the poor noise properties of low-cost sensors can make the robust application of the MEKF a difficult task (Hamel & Mahony, 2006; Mahony et al., 2008). Furthermore, the limited processing resources associated with low budget robotic systems makes the application of more computationally demanding estimation schemes infeasible.

The need for robust attitude estimators for use with low-cost IMUs has led to the development of a class of nonlinear attitude estimators (Grip, Fossen, Johansen, & Saberi, 2012; Hamel & Mahony, 2006; Hua, 2010; Jensen, 2011; Kinsey & Whitcomb, 2007; Mahony, Cha, & Hamel, 2006; Mahony, Hamel, & Pflimlin, 2005; Mahony et al., 2008). These estimators are intuitive, easily applied, and may be implemented using only measurements taken by low-cost IMUs. Moreover, using Lyapunov stability techniques the convergence properties of these estimators can be guaranteed. The structure of the estimator of Mahony et al. (2008) is inspired by the stability analysis of a particular attitude error function, a function that measures the error between an estimated configuration and the true configuration. In fact, nonlinear attitude estimator and control design can be accomplished by first selecting an attitude error function and deriving the structure of the estimator through a Lyapunov stability analysis (Bullo & Lewis, 2005; Lee, 2012). It is unsurprising then that the choice of attitude error function is directly related to the convergence properties of the estimator. The particular attitude error function used in

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Mahony et al. (2005), and its subsequent extensions (Grip et al., 2012; Hamel & Mahony, 2006; Jensen, 2011; Mahony et al., 2006, 2008), has remained unchanged. A drawback of this attitude error function is that the resulting attitude estimator may be slow to converge, particularly when the initial attitude estimation error is large.

The main contribution of this work is the design and characterization of a nonlinear attitude estimator based on an alternate error function. The alternate attitude error function has been previously considered in Zlotnik and Forbes (2015). This paper builds upon the results of Zlotnik and Forbes (2015) by providing a proof of exponential convergence as well as experimental results. The attitude error function selected results in an attitude estimator with faster convergence properties than similar estimators considered in the literature. As the closed-loop control of autonomous vehicles relies on attitude estimates, fast convergence of an attitude estimator is desirable. To achieve a similar convergence rate using the estimators of Hamel and Mahony (2006), Mahony et al. (2006) and Mahony et al. (2005, 2008) the estimator gain must be increased. However, larger estimator gain results in higher amplification of high frequency noise at steady-state. Furthermore, large estimator gain can lead to poor performance when implemented in discrete-time, as is the case in practice. The estimator presented in this paper achieves faster convergence while maintaining equivalent asymptotic behavior to similar estimators. Vector measurements, along with rate-gyro measurements, are used to propagate the attitude estimate as well as an estimate of the rate-gyro bias. Stability results are included that show exponential stability of the desired equilibrium point provided a constraint on the angular velocity is satisfied. In addition, simulation and experimental results that demonstrate the desirable properties of the proposed estimator are included.

2. Problem formulation

2.1. Direction cosine matrices and attitude kinematics

The attitude of one reference frame relative to another reference frame can be globally described by a direction cosine matrix (DCM), $\mathbf{C} \in SO(3)$, where $SO(3) = \{\mathbf{C} \in \mathbb{R}^{3\times 3} \mid \mathbf{C}^{\mathsf{T}}\mathbf{C} = \mathbf{1}, \det(\mathbf{C}) = +1\}$ denotes the special orthogonal group. Consider two frames of reference, denoted $\mathcal{F}_a$ and $\mathcal{F}_b$. The DCM $\mathbf{C}_{ba}$ describes the orientation of $\mathcal{F}_b$ relative to $\mathcal{F}_a$. If $\mathcal{F}_b$ denotes the body-fixed frame of a rigid body, then $\mathbf{C}_{ba}$ describes the orientation of that body relative to $\mathcal{F}_a$. A direction cosine matrix can be parameterized in terms of the axis and angle of Euler's theorem (Hughes, 2004):

$$\begin{aligned}\mathbf{C}_{ba} &= \exp(-\phi\mathbf{a}^{\times}) \\ &= \cos(\phi)\mathbf{1} + (1-\cos(\phi))\mathbf{a}\mathbf{a}^{\mathsf{T}} - \sin(\phi)\mathbf{a}^{\times},\end{aligned} \tag{1}$$

where $\mathbf{a} \in \mathbb{S}^2, \mathbb{S}^2 = \{\mathbf{r} \in \mathbb{R}^3 \mid \sqrt{\mathbf{r}^{\mathsf{T}}\mathbf{r}} = 1\}$, is the axis and $\phi \in \mathbb{R}$ is the angle of rotation about that axis.

Poisson's kinematic equation, commonly referred to as simply Poisson's equation, describes the relationship between the time rate of change of $\mathbf{C}_{ba}$ and angular velocity, namely

$$\dot{\mathbf{C}}_{ba} = -\boldsymbol{\omega}_b^{ba^{\times}}\mathbf{C}_{ba}, \tag{2}$$

where $\boldsymbol{\omega}_b^{ba}$ is the angular velocity of $\mathcal{F}_b$ relative to $\mathcal{F}_a$ (Kane, Likins, & Levinson, 1983). The subscript 'b' indicates that the angular velocity has been resolved in $\mathcal{F}_b$. The operator $(\cdot)^{\times} : \mathbb{R}^3 \to \mathfrak{so}(3)$ maps a 3×1 column matrix to the set of 3×3 skew-symmetric matrices $\mathfrak{so}(3) = \{\mathbf{S} \in \mathbb{R}^{3\times 3} \mid \mathbf{S} + \mathbf{S}^{\mathsf{T}} = \mathbf{0}\}$. For example, if $\mathbf{v} = [v_1\ v_2\ v_3]^{\mathsf{T}} \in \mathbb{R}^3$, then

$$\mathbf{v}^{\times} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}.$$

The operator $(\cdot)^{\vee} : \mathfrak{so}(3) \to \mathbb{R}^3$ is the inverse operation such that $(\mathbf{v}^{\times})^{\vee} = \mathbf{v}, \forall \mathbf{v} \in \mathbb{R}^3$. In addition, the symmetric and anti-symmetric projection operators are defined as $\mathcal{P}_s(\cdot)$ and $\mathcal{P}_a(\cdot)$, respectively, such that $\mathcal{P}_s(\mathbf{U}) = \frac{1}{2}(\mathbf{U} + \mathbf{U}^{\mathsf{T}})$ and $\mathcal{P}_a(\mathbf{U}) = \frac{1}{2}(\mathbf{U} - \mathbf{U}^{\mathsf{T}})$, $\mathbf{U} \in \mathbb{R}^{3\times 3}$. The identities

$$-\mathbf{r}^{\times}\mathbf{s}^{\times} = (\mathbf{r}^{\mathsf{T}}\mathbf{s})\mathbf{1} - \mathbf{s}\mathbf{r}^{\mathsf{T}}, \tag{3}$$

$$(\mathbf{r}^{\times}\mathbf{s})^{\times} = \mathbf{s}\mathbf{r}^{\mathsf{T}} - \mathbf{r}\mathbf{s}^{\mathsf{T}}, \tag{4}$$

$$\mathbf{U}^{\mathsf{T}}\mathbf{r}^{\times} + \mathbf{r}^{\times}\mathbf{U} = ((\mathrm{tr}(\mathbf{U})\mathbf{1} - \mathbf{U})\mathbf{r})^{\times}, \tag{5}$$

$$\frac{1}{2}\mathrm{tr}(\mathbf{U}\mathbf{r}^{\times}) = -\mathbf{r}^{\mathsf{T}}\mathcal{P}_a(\mathbf{U})^{\vee}, \tag{6}$$

$$\mathbf{r}^{\mathsf{T}}\mathbf{s} = -\frac{1}{2}\mathrm{tr}(\mathbf{r}^{\times}\mathbf{s}^{\times}), \tag{7}$$

where $\mathbf{r}, \mathbf{s} \in \mathbb{R}^3$ and $\mathbf{U} \in \mathbb{R}^{3\times 3}$ will prove useful in derivations forthcoming.

2.2. Measurements

Consider a vehicle equipped with sensors capable of measuring a minimum of two reference vectors as well as angular velocity. It is assumed that the angular velocity measurement comes from a rate-gyro corrupted by bias and noise such that

$$\boldsymbol{\omega}_b^{y} = \boldsymbol{\omega}_b^{ba} + \mathbf{b} + \boldsymbol{\mu}^{\omega}, \tag{8}$$

where $\boldsymbol{\omega}_b^{y} \in \mathbb{R}^3$ is the measured angular velocity, $\mathbf{b} \in \mathbb{R}^3$ is an unknown constant bias, and $\boldsymbol{\mu}^{\omega} \in \mathbb{R}^3$ is the sensor noise associated with the rate-gyro.

The vector measurements can be used to construct an instantaneous geometric approximation of $\mathbf{C}_{ba}$. Let $\mathbf{C}_{ba}^{y}$ denote the instantaneous geometric approximation of $\mathbf{C}_{ba}$ and let $\mathbf{y}_b^j, j = 1, \ldots, n$, where n is the number of vector measurements available, denote the vector measurements. There are many methods to construct $\mathbf{C}_{ba}^{y}$ or an equivalent quaternion using $\mathbf{y}_b^j, j = 1, \ldots, n$. These include TRIAD, FOAM, QUEST, ESOQ, ESOQ2, and others (Markley & Crassidis, 2014). The Singular Value Decomposition (SVD) method is one such technique that will be used in this paper. Using the SVD method $\mathbf{C}_{ba}^{y}$ is constructed as (Markley, 1988)

$$\mathbf{C}_{ba}^{y} = \mathbf{V}\mathrm{diag}\{1, 1, \det(\mathbf{V})\det(\mathbf{U})\}\mathbf{U}^{\mathsf{T}}, \tag{9}$$

where $\mathbf{V}$ and $\mathbf{U}$ are orthogonal and, along with the diagonal matrix $\boldsymbol{\Sigma}$, comprise the singular value decomposition of the matrix $\sum_{j=1}^{n} 1/w_j \mathbf{y}_b^j \mathbf{y}_a^{j^{\mathsf{T}}}$ such that

$$\sum_{j=1}^{n} \frac{1}{w_j}\mathbf{y}_b^j\mathbf{y}_a^{j^{\mathsf{T}}} = \mathbf{V}\boldsymbol{\Sigma}\mathbf{U}^{\mathsf{T}}. \tag{10}$$

Constants $w_j, j = 1, \ldots, n$, are strictly positive values associated with the degree of confidence in the measurements. Measurements $\mathbf{y}_b^j, j = 1, \ldots, n$, are typically corrupted by high levels of noise and thus $\mathbf{C}_{ba}^{y}$ may be of poor quality. Additionally, vector measurements may not be available at as high a frequency as, say, rate-gyro measurements. As such, a filter that incorporates rate-gyro data as well as vector measurements to produce a higher quality attitude estimate is sought.

Before moving onto a discussion of the particular attitude estimation algorithm considered in this paper, a brief comment on the special case of IMU measurements is now given. Low-cost IMUs are generally equipped with three sensors: (a) a rate-gyro, (b) an accelerometer, and (c) a magnetometer.

(a) The rate-gyro measures the true angular velocity corrupted by noise and bias, as shown in (8).