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Data-driven design of two degree-of-freedom nonlinear controllers: The D^2-IBC approach[☆]

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ABSTRACT

In this paper, we introduce and discuss the Data-Driven Inversion-Based Control (D^2-IBC) method for nonlinear control system design. The method relies on a two degree-of-freedom architecture, with a nonlinear controller and a linear controller running in parallel, and does not require any detailed physical knowledge of the plant to control. Specifically, we use input/output data to synthesize the controller by employing convex optimization tools. We show the effectiveness of the proposed approach on a benchmark simulation example, regarding control of the Duffing system.

1. Introduction

One of the most natural ways to force the output of a nonlinear system to follow a given trajectory is to employ a feedforward controller described by the inverse of the system dynamics. When fed by the reference trajectory, the output of such a controller will correspond exactly to the desired input of the system, i.e. the input signal producing an output sequence equal to the reference one. Unfortunately, such an ideal controller is not computable and/or applicable in most practical cases. The problems can be many, e.g. the system dynamics is not invertible, there are unstable zero dynamics, the model of the plant is not an accurate description of all the underlying dynamics, the plant is affected by noises/disturbances, or other application-specific issues.

The above observations led – for some classes of systems – to the well-known *feedback linearization* approach (Isidori, 1995; Khalil, 1996), where the objective of the nonlinear controller is milder, i.e. the controller is no longer required to fully invert the system dynamics, but only to linearize it around the current operating point. Nevertheless, such an approach suffers from some critical drawbacks, too. First of all, only input affine systems can typically be dealt with (Isidori, 1995; Khalil, 1996). Secondly, such an approach is still very sensitive to model errors, which may jeopardize the performance but also destabilize the system, when implemented in a real-world setup.

In the last decades, these premises have produced several research directions with the aim to overcome the limits of the above (appealing) approaches for control of nonlinear systems. Among the others, the most common activities in the field are: approximate linearization via feedback (Guardabassi & Savaresi, 2001), feedforward linearization (Hagenmeyer & Delaleau, 2003), robust feedback linearization (Marino & Tomei, 1996), model predictive control (Mayne, Rawlings, Rao, & Scokaert, 2000), identification for control (Gevers, 2005) and direct data-driven control (Novara, Fagiano, & Milanese, 2013; Radac, Precup, Petriu, Preitl, & Dragos, 2013).

Unlike the others, *identification for control* and *direct data-driven control* approaches do not (intrinsically) require an accurate physical model of the system to control, because in the former perspective only the main dynamics of interest for control are accounted for, and in the latter approach the controller is directly computed from experimental measurements.

Concerning *identification for control*, a lot of work has been done to highlight what are the properties a model should have to be suitable for model-based control. In this field, a significant insight has been obtained into what is the most proper experimental setup

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(Hjalmarsson, Gevers, & De Bruyne, 1996) and how robustness should be taken into account in the design of the identification procedure (Gevers, 1994). As far as we are aware, such an analysis has been carried out for linear systems only.

Another interesting change of perspective is that of direct design of controllers from data, which obviously offers a great potential against undesired modeling errors, but also offers new theoretical challenges to the systems and control community. In this field, a lot of work has already been done, ranging from the Ziegler and Nichols method (Ziegler & Nichols, 1942) to Virtual Reference Feedback Tuning (VRFT Campi, Lecchini, & Savaresi, 2002; Formentin, Savaresi, & Del Re, 2012) until the recent data-driven loop-shaping approach (Formentin & Karimi, 2013). However, only few contributions have focused on nonlinear systems. Among these, *neural networks* have played a major role, see e.g. Polycarpou (1996) and Yeşildirek and Lewis (1995). Notwithstanding their evident general applicability, these methods are hard to implement from a practical point of view, due to the lack of criteria for optimal network selection. Since the related optimization problems are typically non-convex, there is also no guarantee that the resulting controller corresponds to the optimal one.

The nonlinear version of the *VRFT method* (Campi & Savaresi, 2006) represents an effective tool to make the nonlinear system behave like a desired linear reference model, using convex optimization only. Although the idea behind the method sounds natural and appealing, the approach in the current form is not able to guarantee a certain level of performance, when the system moves far from the input trajectory of the identification data set. Instead, with the recent *Direct Feedback approach* (DFK Novara et al., 2013), a stabilizing controller can be computed, which can also guarantee that the norm of the tracking error is bounded, under some assumptions on the identification data set and solving only convex problems. Specifically, the DFK controller aims to be the data-driven counterpart of the model-driven inversion-based controller, but without needing any assumption on the model parametrization. However, the DFK approach may not be suitable for the control of systems described by a regression function that is non-invertible (non-injective) with respect to the command input (this limitation is common to many methods, including feedback linearization). Also, DFK is based on full-state feedback and its extension to the output feedback case has not been systematically developed yet.

In this paper, we propose a novel approach called *Data-Driven Inversion-Based Control* (D^2-IBC), which exploits the main ideas in identification for control and direct data-driven controller tuning. The method is developed within the same mathematical framework of DFK, but without the above limitations and with enhanced tracking performance guarantees. More specifically, the main innovative features of this approach are as follows: (i) D^2-IBC relies on a two degree-of-freedom architecture, composed by a nonlinear controller and a linear controller in parallel, allowing both compensation of nonlinearities and performance boosting; (ii) it extends the identification for control rationale to nonlinear systems: here, a nonlinear model is selected depending on its suitability for control design, whereas the matching of the open-loop dynamics is considered less important;[2] (iii) a novel nonlinear control design method based on the above identified model is used to design the nonlinear control block; this method, called Nonlinear Inversion Control (NIC), was (partially) published in the technical report (Novara & Milanese, 2014);[3] (iv) since the design of the linear controller involves the dynamics of an unknown input sensitivity function, a direct data-driven method is used to design the linear control block: the method is the well known VRFT, but it is adapted to the architecture at hand.

We should remark that, in this work, we focus only on SISO systems. The MIMO extension is not straightforward and is object of current research.

The remaining part of the paper is as follows. The general principles behind the D^2-IBC approach are discussed in Section 2, where also a theoretical result is given, motivating the proposed two degree-of-freedom architecture. The specific design algorithms are presented in Section 3, while their effectiveness is assessed on a benchmark simulation example in Section 4. A comparison with the state of the art techniques and some concluding remarks are given in Section 5.

Notation. A column vector $x \in \mathbb{R}^{n_x \times 1}$ is denoted as $x = (x_1, \ldots, x_{n_x})$. A row vector $x \in \mathbb{R}^{1 \times n_x}$ is denoted as $x = [x_1, \ldots, x_{n_x}] = (x_1, \ldots, x_{n_x})^\top$, where $\top$ indicates the transpose.

A discrete-time signal (i.e. a sequence of vectors) is denoted with the bold style: $\boldsymbol{x} = (x_1, x_2, \ldots)$, where $x_t \in \mathbb{R}^{n_x \times 1}$ and $t = 1, 2, \ldots$ indicates the discrete time; $x_{i,t}$ is the ith component of the signal $\boldsymbol{x}$ at time t.

A regressor, i.e. a vector that, at time t, contains n present and past values of a variable, is indicated with the bold style and the time index: $\boldsymbol{x}_t = (x_t, \ldots, x_{t-n+1})$.

The ℓ_p norms of a vector $x = (x_1, \ldots, x_{n_x})$ are defined as

$$\|x\|_p \doteq \begin{cases} \left(\sum_{i=1}^{n_x} |x_i|^p \right)^{\frac{1}{p}}, & p < \infty, \\ \max_i |x_i|, & p = \infty. \end{cases}$$

The ℓ_p norms of a signal $\boldsymbol{x} = (x_1, x_2, \ldots)$ are defined as

$$\|\boldsymbol{x}\|_p \doteq \begin{cases} \left(\sum_{t=1}^{\infty} \sum_{i=1}^{n_x} \left|x_{i,t}\right|^p \right)^{\frac{1}{p}}, & p < \infty, \\ \max_{i,t} \left|x_{i,t}\right|, & p = \infty, \end{cases}$$

where $x_{i,t}$ is the ith component of the signal $\boldsymbol{x}$ at time t. These norms give rise to the well-known ℓ_p Banach spaces.

2. The D^2-IBC approach

2.1. Problem setting

Consider a nonlinear discrete-time SISO system in regression form:

$$\begin{aligned} y_{t+1} &= g\left(\boldsymbol{y}_t, \boldsymbol{u}_t, \boldsymbol{\xi}_t\right) \\ \boldsymbol{y}_t &= (y_t, \ldots, y_{t-n+1}) \\ \boldsymbol{u}_t &= (u_t, \ldots, u_{t-n+1}) \\ \boldsymbol{\xi}_t &= (\xi_t, \ldots, \xi_{t-n+1}) \end{aligned} \qquad (1)$$

where $u_t \in U \subset \mathbb{R}$ is the input, $y_t \in \mathbb{R}$ is the output, $\xi_t \in \Xi \doteq \left[-\bar{\xi}, \bar{\xi}\right]^{n_\xi} \subset \mathbb{R}^{n_\xi}$ is a disturbance including both process and measurement noises, and n is the system order. U and Ξ are compact sets. In particular, $U \doteq [\underline{u}, \overline{u}]$ accounts for input saturation.

Suppose that the system (1) is unknown, but a set of measurements is available:

$$\mathcal{D} \doteq \left\{\tilde{u}_t, \tilde{y}_t\right\}_{t=1-L}^{0} \qquad (2)$$

where $\tilde{u}_t \in U, \tilde{y}_t \in Y, Y = [-\bar{y}, \bar{y}]$ and $\bar{y} \doteq \max_t |\tilde{y}_t| < \infty$. The tilde is used to indicate the input and output samples of the data

2 Notice that – concerning the identification part – this contribution is different from that of Novara et al. (2013), where the open-loop dynamics is skipped and the controller is directly identified from data.

3 This approach shares some features with nonlinear Model Predictive Control (MPC). The main differences between the two approaches will be discussed at the end of the paper.