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Robust self-triggered MPC for constrained linear systems: A tube-based approach ☆

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ABSTRACT

We propose a robust self-triggered control algorithm for constrained linear discrete-time systems subject to additive disturbances based on MPC. At every sampling instant, the controller provides both the next sampling instant, as well as the inputs that are applied to the system until the next sampling instant. By maximizing the inter-sampling time subject to bounds on the MPC value function, the average sampling frequency in the closed-loop system is decreased while guaranteeing an upper bound on the performance loss when compared with an MPC scheme sampling at every point in time. Robust constraint satisfaction is achieved by tightening input and state constraints based on a Tube MPC approach. Moreover, a compact set in the state space, which is a parameter in the MPC scheme, is shown to be robustly asymptotically stabilized.

1. Introduction

For control systems where the communication between system and controller constitutes a considerable effort in terms of energy or infrastructure, the performance of the control system must be weighed against the amount of communication necessary to achieve this performance. In this context, it has been found that controllers with aperiodic scheduling of input and measurement updates may achieve a better trade-off between performance and overall communication load than controllers with periodic scheduling, see for example Heemels, Johansson, and Tabuada (2012) and You and Xie (2013) and the references therein. In particular, *event-triggered* and *self-triggered* control schemes have been proposed, where in the first class of controllers a new input is computed and communicated to the system only if certain conditions on the state of the system are met (defining an "event"), and in the second class the next update time is calculated explicitly at the current update time based on the current state of the system. The main difference between the two classes of controllers is that event-triggered control requires periodic or continuous measurement of the system state (or output) while in self-triggered control the sensors may be shut down completely between updates. For a recent overview of event-triggered and self-triggered control we refer the interested reader to Heemels et al. (2012). While self-triggered control schemes have the advantage of requiring overall less information from the system in general, this advantage at the same time makes these schemes more susceptible to disturbances and uncertainties when compared to event-triggered control schemes.

In this paper, we present a robust self-triggered MPC method based on ideas from Tube MPC (Chisci, Rossiter, & Zappa, 2001; Langson, Chryssochoos, Raković, & Mayne, 2004). MPC is a control method where the control input at each sampling instant is defined as the first part of the solution of a finite-horizon optimal control problem. MPC is especially suited for setups with hard constraints on the input and states, as these constraints can be explicitly taken into account in the definition of the optimization problem. For an overview of MPC, please refer to Mayne (2014), Mayne, Rawlings, Rao, and Scokaert (2000) and Rawlings and Mayne (2009). For linear time-invariant systems subject to bounded additive disturbances, Tube MPC has proven to be an effective way of robustifying MPC. Tube MPC is based on set-valued predictions of the state and input of the system taking the effect of the disturbances into account. A key ingredient in Tube MPC is the assumption that *feedback* is present at every point in time, reducing the effect of the disturbances and thereby restricting the growth of the predicted sets (Chisci et al., 2001). In the present work, the

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assumption of feedback at every point in time is not satisfied as we are explicitly designing controllers with extended periods of open-loop control. This limitation leads to a stronger growth of the uncertainty in the prediction. However, it holds that feedback will be present at *some* time in the future which still restricts the growth of the uncertainty in the predictions, as we will see. This knowledge is used in the construction of the tightened constraint sets, which extend those employed in Chisci et al. (2001). An additional challenge when adapting Tube MPC methods to a self-triggered setup is the fact that the asymptotic bound on the system state depends on the times between control updates, which are determined online. In standard Tube MPC, the times between control updates are uniform. We address this problem by carefully designing the MPC cost function and determining the times between control updates according to the evolution of this function. This enables us to provide an offline a priori asymptotic bound on the system state which is also a tuning parameter of the MPC scheme. Note that in Eqtami (2013), and the references therein, robust event- and self-triggered MPC schemes are proposed based on tubes where *no* feedback is assumed in the predictions. For open-loop unstable systems this has the drawback of leading to an exponential growth of the predicted uncertainty, thereby imposing an upper bound on the maximal prediction horizon if state constraints are present and reducing the feasible region of the MPC scheme. Inspired by Barradas Berglind, Gommans, and Heemels (2012) and Gommans, Antunes, Donkers, Tabuada, and Heemels (2014), the self-triggered controller in the present paper maximizes, at each sampling instant, the time until the next sampling instant subject to constraints on the associated MPC cost function and addresses the mentioned issue of exponentially growing uncertainty under open-loop predictions. These constraints on the MPC cost will enable us to prove that the cost of our new self-triggered MPC scheme is bounded by the cost associated with the solution of a standard periodically triggered MPC scheme multiplied by a positive factor which is a tuning knob of our scheme. Another tuning knob is the size of the set that is robustly stabilized. As a consequence, the proposed self-triggered MPC scheme allows trade-offs between closed-loop performance, the asymptotic bound on the system state, and the average communication rate.

Alternative MPC-based self-triggered control schemes are available. In Henriksson (2014) and Henriksson, Quevedo, Sandberg, and Johansson (2012), an MPC scheme for undisturbed systems is considered, where the sampling rate is part of the MPC cost function. In Antunes and Heemels (2014), an optimization-based scheme is proposed where at each sampling instant the input is decided by selecting an optimal scheduling *sequence* with respect to a quadratic cost function. Note that both Barradas Berglind et al. (2012), Henriksson (2014) and Henriksson et al. (2012) do not consider disturbances, while Antunes and Heemels (2014) and Gommans et al. (2014) consider disturbances but no constraints on the state or input. In Kögel and Findeisen (2014), a self-triggered scheme for disturbed systems under constraints was presented based on robust control-invariant sets. However, neither stability, nor performance is addressed. In earlier results on self-triggered MPC for disturbed systems (Aydiner, 2014; Brunner, Heemels, & Allgöwer, 2014), the asymptotic bound depended on the optimal MPC cost function, which is usually not easily obtainable. In Aydiner, Brunner, Heemels, and Allgöwer (2015), a robust self-triggered MPC scheme based on Raković, Kouvaritakis, Findeisen, and Cannon (2012) was presented, which allows a similar a priori determination of the asymptotic bound, while employing a conceptionally different way of describing the uncertainties in the prediction. The MPC schemes proposed in Eqtami (2013) allow an a priori determination of the guaranteed asymptotic bound in the form of an ellipsoidal set, which is a conservative restriction for the linear systems considered in the present paper.

The remainder of the paper is structured in the following way. Some notes on notation and some preliminary results and definitions are given in Section 2. The problem setup is stated in Section 3. In Section 4, a Tube MPC optimization problem is defined, where the first steps in the prediction horizon are assumed to be applied in an open-loop fashion. The main results of the paper are given in Section 5, where the robust self-triggered scheme and its properties are presented. Section 6 contains some notes on the implementation and the complexity of the algorithm. Section 7 concludes the paper.

For the sake of readability, most of the proofs are located in the Appendix.

2. Notation and preliminaries

Let $\mathbb{N}$ denote the set of non-negative integers. For $q, s \in \mathbb{N} \cup \{\infty\}$, let $\mathbb{N}_{\geq q}$ and $\mathbb{N}_{[q,s]}$ denote the sets $\{r \in \mathbb{N} \mid r \geq q\}$ and $\{r \in \mathbb{N} \mid q \leq r \leq s\}$, respectively. The set of non-negative real numbers is denoted by $\mathbb{R}_+$. For $n \in \mathbb{N}$, I_n denotes the $n \times n$ identity matrix. A matrix with zero entries is denoted by 0, where the dimension is defined by context. Given sets $\mathcal{X}, \mathcal{Y} \subseteq \mathbb{R}^n$, a scalar α, and matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times m}$ we define $\alpha\mathcal{X} := \{\alpha x \mid x \in \mathcal{X}\}$, $A\mathcal{X} := \{Ax \mid x \in \mathcal{X}\}$, and $B^{-1}\mathcal{X} := \{x \in \mathbb{R}^m \mid Bx \in \mathcal{X}\}$. The Minkowski set addition is defined by $\mathcal{X} \oplus \mathcal{Y} := \{x + y \mid x \in \mathcal{X}, y \in \mathcal{Y}\}$. Given a vector $x \in \mathbb{R}^n$ we define $\mathcal{X} \oplus x := x \oplus \mathcal{X} := \{x\} \oplus \mathcal{X}$. The Pontryagin set difference (Kolmanovsky & Gilbert, 1995, 1998) is defined by $\mathcal{X} \ominus \mathcal{Y} := \{z \in \mathbb{R}^n \mid z \oplus \mathcal{Y} \subseteq \mathcal{X}\}$. Given a (finite or infinite) sequence of sets $\mathcal{X}_i$ for $i \in \mathbb{N}_{[a,b]}$ with $a \in \mathbb{N}$ and $b \in \mathbb{N} \cup \{\infty\}$, we define $\bigoplus_{i=a}^{b} \mathcal{X}_i := \left\{\sum_{i=a}^{b} x_i \mid x_i \in \mathcal{X}_i\right\}$. By convention, the empty sum is equal to $\{0\}$. Similarly, for any vectors $v_i \in \mathbb{R}^n$, $i \in \mathbb{N}$, we define $\sum_{i=a}^{b} v_i = 0$ for any $a, b \in \mathbb{N}$ if $a > b$. We call a compact, convex set containing the origin a C-set. A C-set containing the origin in its (non-empty) interior is called a PC-set. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$. If additionally $\alpha(s) \to \infty$ as $s \to \infty$, α is said to belong to class $\mathcal{K}_\infty$. The Euclidean norm of a vector $v \in \mathbb{R}^n$ is denoted by $|v|$. Given any compact set $\mathbb{S} \subseteq \mathbb{R}^n$, the distance between v and $\mathbb{S}$ is defined by $|v|_{\mathbb{S}} := \min_{s\in\mathbb{S}} |v - s|$. The convex hull of a set $\mathcal{X} \subseteq \mathbb{R}^n$ is denoted by $\mathrm{convh}(\mathcal{X})$. Define finally the Euclidean unit ball by $\mathcal{B} := \{x \in \mathbb{R}^n \mid |x| \leq 1\}$.

Lemma 1. *Let $\mathcal{X}, \mathcal{Y}, \mathcal{Z} \subseteq \mathbb{R}^n$ be compact convex sets. Let further $A \in \mathbb{R}^{m\times n}$. Then it holds that $\mathcal{X} \oplus \mathcal{Y} = \mathcal{Y} \oplus \mathcal{X}$, $\mathcal{X} \ominus (\mathcal{Y} \oplus \mathcal{Z}) = (\mathcal{X} \ominus \mathcal{Y}) \ominus \mathcal{Z}$, $(\mathcal{X} \oplus \mathcal{Y}) \ominus \mathcal{Y} = \mathcal{X}$, $(\mathcal{X} \ominus \mathcal{Y}) \oplus \mathcal{Y} \subseteq \mathcal{X}$, $A(\mathcal{X} \oplus \mathcal{Y}) = A\mathcal{X} \oplus A\mathcal{Y}$, and $A(\mathcal{X} \ominus \mathcal{Y}) \subseteq (A\mathcal{X} \ominus A\mathcal{Y})$.*

Next, we define stability properties of dynamical systems subject to disturbances of the form

$$(x_{k+1}^\top, z_{k+1}^\top)^\top = f(x_k, z_k, w_k), \tag{1}$$

where $f : \mathbb{R}^n \times \mathbb{R}^p \times \mathcal{W} \to \mathbb{R}^n$, $k \in \mathbb{N}$, are given, and $x_k \in \mathbb{R}^n$ and $w_k \in \mathcal{W} \subseteq \mathbb{R}^n$, are the state and disturbance at time $k \in \mathbb{N}$, and $z_k \in \mathbb{R}^p$ is an internal state of the controller with $z_0 = 0$.

Definition 2 (*Robust Lyapunov Stability of Sets*)**.** A set $\mathcal{Y} \subseteq \mathbb{R}^n$ is *robustly Lyapunov stable* for System (1) if there exist a $\mathcal{K}$-function γ and a $\delta > 0$ such that for any initial condition $x_0 \in \{x \in \mathbb{R}^n \mid |x|_{\mathcal{Y}} \leq \delta\}$ and any disturbances with $w_k \in \mathcal{W}$, $k \in \mathbb{N}$, it holds that $|x_k|_{\mathcal{Y}} \leq \gamma(|x_0|_{\mathcal{Y}})$ for all $k \in \mathbb{N}$.

Definition 3 (*Robust Asymptotic Stability of Sets*)**.** A set $\mathcal{Y} \subseteq \mathbb{R}^n$ is *robustly asymptotically stable* for System (1) with $\hat{\mathcal{X}} \subseteq \mathbb{R}^n$ *belonging to its region of attraction* if it is robustly Lyapunov stable for this system and $\lim_{k\to\infty} |x_k|_{\mathcal{Y}} = 0$ for all $x_0 \in \hat{\mathcal{X}}$, and any disturbances with $w_k \in \mathcal{W}$, $k \in \mathbb{N}$.