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A distributed simultaneous perturbation approach for large-scale dynamic optimization problems[☆]

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A B S T R A C T

We consider distributed optimization problems of large-scale dynamic systems where the global cost is the sum of all individual costs of subsystems which are only known to the associated agent. To solve this problem, a distributed simultaneous perturbation approach (D-SPA) is proposed based on simultaneous perturbation techniques as well as consensus theory. The proposed method is model-free so long as all individual costs can be measured and requires little knowledge on the coupling structure of the problem to be optimized. The convergence of the proposed scheme is proved using singular perturbation and averaging theory. In particular, with proper choice of the parameters under design, we show that the proposed scheme is able to converge to the neighborhood of the Pareto optimum of the problem so long as the energy of perturbation signals is sufficiently small. Moreover, the proposed approach is applied to a simulated offshore wind farm for energy maximization and a comprehensive comparison with the existing state-of-the-art technique is made to illustrate its effectiveness.

1. Introduction

Distributed optimization has recently been receiving much attention due to its wide applications in areas such as formation control, resource allocation and wireless communication, to name a few. This technique is especially suitable for large-scale problems as it only requires local resources (e.g., local sensing, local communication and local control in Networked Control Systems Baillieul & Antsaklis, 2007) for achieving global results.

In the existing literature, gradient-based methods are widely employed to solve large-scale optimization problems in a distributed way. In particular, Tsitsiklis, Bertsekas, and Athans (1986) first studied the distributed gradient-like optimization algorithm in which a bunch of processors perform computations and exchange messages asynchronously intending to minimize a certain cost function. In the context of distributed computation, consensus mechanism lends itself to distributed implementation of algorithms as it allows agents to obtain global results by taking actions asynchronously and communicating limited information with its neighbors even over varying communication topology (Olfati-Saber, Fax, & Murray, 2007). In line with these works, Nedic and Ozdaglar (2009) applied consensus theory to multi-agent optimization problems where each agent only knows its cost, resulting in distributed sub-gradient methods which can accommodate varying communication topology. Zanella, Varagnolo, Cenedese, Pillonetto, and Schenato (2011) solved the same unconstrained optimization problem by distributing the conventional Newton–Raphson algorithm using consensus-like strategies. Two drawbacks of these kinds of methods are that the communication cost will increase with the dimension of the problem to be optimized and the gradient should be computable exactly for optimization. Dual decomposition has also been used to solve large-scale optimization problems (Bertsekas, Nedic, & Ozdaglar, 2003). Rather than directly dealing with the primal problem, this method solves the dual counterpart which can be further divided into several small sub-problems that are relatively easy to solve. Examples include formation control (Raffard, Tomlin, & Boyd, 2004), multi-agent optimization (Terelius, Topcu, & Murray, 2011), network utility maximization (Low & Lapsley, 1999) and resource allocation (Xiao, Johansson, & Boyd, 2004). This technique, however, requires the cost function to be separable for efficient gradient calculation and needs to consider the specific coupling topology to decouple the problem.

For problems in which gradients may not be available, we have to resort to some gradient approximation approaches.

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Instead of using the true gradient, one can solve the optimization problem by using the pseudo-gradient estimated from probing the system using perturbation techniques. One promising approach is extremum seeking control (ESC) which has been widely employed to optimize systems without knowing their specific reference-to-output equilibrium map (Ariyur & Krstic, 2003). In this work, we are particularly interested in the distributed implementation of ESC, termed as D-ESC. Existing applications of D-ESC include mobile sensor networks (Stankovic, Johansson, & Stipanovic, 2012) and non-cooperative game (Frihauf, Krstic, & Basar, 2012). Since they only considered non-cooperative games, their results are of Nash Equilibrium (Basar & Olsder, 1999), which is a sub-optimal solution. In order to obtain the global optimum (i.e., Pareto-efficient solution), Kvaternik and Pavel (2012) incorporated consensus protocols into extremum seeking algorithms. However, there is no explicit explanation on how to obtain the gradient information using certain probing technique which is crucial for implementation. In our previous work, we have also developed a preliminary version of D-ESC scheme which takes into account the constraints but the proposed scheme needs to explicitly consider the physical interaction topology among agents which is not practical especially in dynamically changing environment (Xu & Soh, 2013). Though extremum seeking control has a long history (Tan, Moase, Manzie, Nešić, & Mareels, 2010), the rigorous proof of its stability of the general form is given only recently by Krstic and Wang (2000) for local results and Tan, Nešić, and Mareels (2006) for non-local results using singular perturbation and averaging analysis. It is claimed that their stability results can be extended without much effort to multi-variable extremum seeking control as done in Rotea (2000).

In this paper, we propose a new approach for solving large-scale dynamic optimization problems by resorting to simultaneous perturbation (Spall, 1998; Zak et al., 2004) and consensus theory. In particular, we consider a distributed optimization problem where agents are collaborating to seek the optimum of the sum of individual costs which can be measured and known only by the associated agent. In this approach, each agent is assumed to update only a subset of the components of the global vector, which is desirable in cases where only local action can be taken. The proposed scheme, termed as distributed simultaneous perturbation approach (D-SPA), is model-free (derivative-free) and, different from most existing literature, only requires little knowledge regarding the dimension of the system as well as the underlying coupling structure of the problem. Thus, our scheme has the potential to adapt to changing environments so long as it is slow enough. In addition, it is envisioned that the favorable properties of consensus algorithms are preserved, such as allowing for asynchronous implementation. In each iteration of the algorithm, regardless of the dimension of the problem, only little measurement data is transmitted for coordination. Moreover, we will show that the D-SPA scheme is able to obtain Pareto-optimum, which takes into account the interest of the adversary, in a distributed manner with a gap of the same order of the root mean square (RMS) amplitude of perturbation signals. In all, the D-SPA scheme is especially suitable for problems where we do not have much knowledge, e.g., wind farm system where the aerodynamic interactions among turbines are difficult to model. However, the drawback of gradient-free techniques is their slow convergence speed. Some extensions can be made to overcome this issue, e.g., Newton-based multi-variable extremum seeking control (Ghaffari, Krstic, & Nešić, 2012).

Notation. We denote by x_i the ith component of a vector x. A variable x without subscript, unless stated otherwise, is viewed as the collection of x_i and written as $x = [x_1, x_2, \dots, x_n]^T$. In addition, we use $\mathbf{1}$ to denote the all-ones column vector with proper dimension, $\odot$ the Hadamard(component-wise) product, $\otimes$ the Kronecker product, $\circ$ the composition of functions, $\mathcal{R}_{\geq 0}$ the set of non-negative reals and $\|\cdot\|$ the Euclidean norm of vectors and the induced norm of matrices. Moreover, we say an error e of certain dimension is of order $O(\varepsilon)$ if $\|e\| \leq k\varepsilon$, where k is some positive constant and ε is a small positive scalar. A continuous function $\gamma : \mathcal{R}_{\geq 0} \to \mathcal{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is strictly increasing and $\gamma(0) = 0$. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and $\gamma(r) \to \infty$ as $r \to \infty$. A continuous function $\kappa : \mathcal{R}_{\geq 0} \times \mathcal{R}_{\geq 0} \to \mathcal{R}_{\geq 0}$ is of class $\mathcal{KL}$ if, for each $s \geq 0$, $\kappa(r, s)$ is of class $\mathcal{K}$ w.r.t. r and, for each $r \geq 0$, $\kappa(r, s)$ is decreasing w.r.t. s and goes to zero as $s \to \infty$. A function is said to be of class $\mathcal{C}^k$ if all of its partial derivatives to k orders exist and continuous. Whenever the context is clear, we may suppress the arguments of a function for compact expression.

2. Problem setting

2.1. Preliminaries

To facilitate the analysis in the sequel, we give the formal definitions of Nash Equilibrium (Basar & Olsder, 1999) and Pareto-optimum (Marler & Arora, 2004) for an N-player nonzero-sum game (Θ, J) where $\Theta = \Theta_1 \times \Theta_2 \cdots \times \Theta_N$ is the set of strategy profiles with Θ_i denoting the strategy set for player $i \in \mathcal{V} := \{1, 2, \dots, N\}$ and $J = [J_1(\theta), J_2(\theta), \dots, J_N(\theta)]^T$ is the cost function for $\theta \in \Theta$.

Definition 1 (*Nash Equilibrium*). A strategy profile $\theta^* = [\theta_1^*, \theta_2^*, \dots, \theta_N^*]^T \in \Theta$ with $\theta_i^* \in \Theta_i$, $i \in \mathcal{V}$ is said to constitute a Nash equilibrium solution for an N-player nonzero-sum game if the following conditions hold

$$J_i(\theta_i, \theta_{-i}^*) \geq J_i(\theta_i^*, \theta_{-i}^*), \quad \forall \theta_i \in \Theta_i,\ i \in \mathcal{V}$$

where θ_{-i}^* denotes the strategies of all other players.

Definition 2 (*Pareto-Optimum*). A strategy profile $\theta^* = [\theta_1^*, \theta_2^*, \dots, \theta_N^*]^T \in \Theta$ with $\theta_i^* \in \Theta_i, i \in \mathcal{V}$ is said to constitute a Pareto-optimal solution for an N-player nonzero-sum game if there does not exist another strategy $\theta \in \Theta$ such that $J_i(\theta^*) \geq J_i(\theta),\ \forall i \in \mathcal{V}$ and $J_k(\theta^*) > J_k(\theta)$ for at least one player $k \in \mathcal{V}$.

Remark 3. The most common way to obtain the Pareto-optimal solution is using the weighted sum method, i.e., minimizing $\sum_{i=1}^N w_i J_i(\theta)$, which admits a unique solution and is sufficient for achieving Pareto optimality (Marler & Arora, 2004). In addition, if one player can losslessly transfer part of its cost to another player (e.g., they have a common currency to evaluate their cost), then we can simply optimize their sum for Pareto-optimality.

Definition 4 (*USPAS (Nešić & Teel, 2001)*). The parameterized system $\dot{x} = f(t, x, \varepsilon)$ is said to be uniformly semi-globally practically asymptotically stable[2] (USPAS) on ε if there exists $\kappa \in \mathcal{KL}$ and, for each pair of strictly positive numbers (Δ, δ), there exists a real number $\varepsilon^* = \varepsilon^*(\Delta, \delta) > 0$ such that for all initial condition x_0 with $\|x_0\| \in \Delta$ and for each $\varepsilon \in (0, \varepsilon^*)$, we have $\|x(t)\| \leq \kappa(\|x_0\|, t - t_0) + \delta, \forall t \geq t_0 \geq 0$.

[2] In the sequel, the stability of a given dynamic system is stated with respect to its equilibrium.