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Brief paper

# Variance-constrained $H_\infty$ control for a class of nonlinear stochastic discrete time-varying systems: The event-triggered design[☆]

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ABSTRACT

In this paper, a general event-triggered framework is set up to deal with the variance-constrained $H_\infty$ control problem for a class of discrete time-varying systems with randomly occurring saturations, stochastic nonlinearities and state-multiplicative noises. Based on the relative error with respect to the measurement signal, an event indicator variable is introduced and the corresponding event-triggered scheme is proposed in order to determine whether the measurement output is transmitted to the controller or not. The stochastic nonlinearities under consideration are characterized by statistical means which can cover several classes of well-studied nonlinearities. A set of unrelated random variables is exploited to govern the phenomena of randomly occurring saturations, stochastic nonlinearities and state-dependent noises. The purpose of the addressed multiobjective control problem is to design a set of time-varying output feedback controller such that, over a finite horizon, the closed-loop system achieves both the prescribed $H_\infty$ noise attenuation level and the state covariance constraints. A recursive matrix inequality approach is developed to derive the sufficient conditions for the existence of the desired finite-horizon controllers, and the analytical characterization of such controllers is also given. Simulation studies are conducted to demonstrate the effectiveness of the developed controller design scheme.

## 1. Introduction

In the past few decades, there has been a surge of research interest in the stochastic control problem since stochastic modeling has been successfully applied in many fields. A large body of literature has been devoted to the stochastic control or filtering problem for different systems such as polynomial stochastic systems (Basin & Hernandez-Gonzalez, 2016; Basin & Rodriguez-Ramirez, 2014; Hernandez-Gonzalez & Basin, 2015), Markovian jumping systems (Wu, Yao, & Zheng, 2012), switched stochastic systems (Wu, Zheng, & Gao, 2013), discrete-time stochastic systems with state-dependent noises (Zhang, Huang, & Zhang, 2007), nonlinear stochastic systems (Dong, Wang, Ding, & Gao, 2015; Niu, Ho, & Wang, 2008) and stochastic sampled-data control system (Shen, Wang, & Huang, 2016). Among various stochastic control schemes, the covariance control (CC) theory has gained particular research attention due primarily to the fact that the performance requirements of many engineering control systems are naturally expressed as the upper bounds on the steady-state variances (Wang, Lam, Ma, Bo, & Guo, 2011). It has been shown that the CC approach is ideally suited to handle the multiobjective design problems where the multiple objectives include, but are not limited to, variance constraints, $H_2$-norm specification, $H_\infty$ performance index and pole placement (Ma, Wang, & Bo, 2015). The CC theory was originally developed for linear systems and has been recently extended to nonlinear stochastic systems (Wang et al., 2011). It is worth pointing out that most results concerning the CC theory have focused on the steady-state behaviors for time-invariant systems over an infinite horizon. However, virtually almost all real-time control processes are time-varying especially when the noise inputs are nonstationary (Shen, Wang, Shu, & Wei, 2010; Wang, Dong, Shen, & Gao, 2013). In such cases, it would make

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more sense to consider the covariance control problems for time-varying systems over a finite-horizon in order to provide a better transient performance.

Due to physical and safety constraints, the sensor saturation is probably one of the most commonly encountered phenomena in practical control systems that can severely degrade the system performance or even lead to unstable behaviors. So far, considerable research attention has been paid to the filtering and control problems for systems with sensor saturation, see Xiao, Cao, and Lin (2004), Zhang, Boukas, and Haidar (2008) and the references therein, where the saturation has been implicitly assumed to occur definitely, i.e., the sensor always undergoes saturation. Such an assumption, however, is not always true. For example, in a networked environment, the sensor saturations may occur in a probabilistic way where the saturation amplitude/intensity may be randomly changeable. Such kind of randomly occurring sensor saturation (ROSS) may result from networked-induced intermittent sensor failures, sensor aging or sudden environment changes. On the other hand, stochastic nonlinearities are often found in networked control systems where the nonlinearities are induced by randomly fluctuated network loads due mainly to the communication limitations. Up to now, some initial efforts have been made on the filtering and control problems for systems with stochastic nonlinearities (Shen et al., 2010), while most available results have been concerned with additive noises only. Note that many plants may be modeled by systems with state-dependent noises and some characteristics of nonlinear systems can be closely approximated by models with state-multiplicative noises rather than by linearized models. It is, therefore, one of the motivations of the present research to investigate how the phenomena of ROSS, stochastic nonlinearities and state-multiplicative noises influence the state variance and $H_\infty$ performance of a class of time-varying control systems.

In networked control systems, an important issue is how to transmit signals more effectively by utilizing the available but limited network bandwidth. To alleviate the unnecessary waste of communication/computation resources that often occurs in conventional time-triggered signal transmissions, a recently popular communication schedule called event-triggered strategy has been proposed in Ding, Wang, and Shen (2015), Ding, Wang, Shen, and Dong (2015), Dong, Wang, Alsaadi, and Ahmad (2015), Peng and Yang (2013) and Yue, Tian, and Han (2013). The triggering mechanism refers to the situation where the measurement output is transmitted to a remote controller/filter only when certain conditions are satisfied. In other words, a constant measurement signal is maintained until a specified event condition is violated in an event generator. In comparison with the conventional time-triggered communication, a notable advantage of the event triggering scheme is its capability of reducing redundant transmissions while preserving the guaranteed system performance. In recent years, increasing attention has been drawn on the event-triggered techniques for stochastic systems and many important results have been reported in the literature, see Hu, Yue, Xie, and Du (2015) and Molin and Hirche (2010). However, it should be pointed out that, many established results referring to event-triggering schemes are in the framework of continuous-time systems and, when it comes to the discrete-time systems, the corresponding results have been scattered. The representative one (Molin & Hirche, 2010) has addressed the problem of designing optimal event-triggered controllers under costly observations, in which the optimal event-trigger depends on the difference between the state estimates at controller and event-trigger. Although the importance of relative error-based event-triggering criterion has been widely recognized, the corresponding results for discrete-time systems have been very few especially when the variance-constrained $H_\infty$ control problem becomes a research focus. It is also noticed that, despite its engineering significance, the event-triggered control problem for *time-varying* stochastic systems with *both variance and $H_\infty$ performance constraints* over a *finite horizon* has not received adequate research attention yet, not to mention the case when *ROSS, stochastic nonlinearities and state-multiplicative noises* are simultaneously present. Therefore, the main motivation of this paper is to shorten such a gap by launching a systematic investigation.

In this paper, we make the first of the few attempts to consider the event-triggered multiobjective control problems for time-varying systems with ROSS, stochastic nonlinearities and state-multiplicative noise. The multiple objectives include the state variance constraints and the $H_\infty$ disturbance rejection attenuation level. *The main contributions of this paper are highlighted as follows.* (1) *A variance-constrained $H_\infty$ controller is proposed for discrete nonlinear time-varying systems in the framework of event-based communication protocol.* (2) *A new event indicator variable is constructed to reflect the event-triggered information in the controller analysis so as to decrease the frequency of data transmission and also reduce the conservatism in the controller design as compared to existing literature.* (3) *The developed recursive algorithm merits online applications.* (4) *The system model addressed is new, which is quite comprehensive to cover time-varying parameters, stochastic nonlinearities, multiplicative noise as well as ROSS, hence reflecting the reality more closely.*

**Notation**. The notation used here is standard except where otherwise stated. tr($A$) represents the trace of a matrix $A$. The symbol $\otimes$ denotes the Kronecker product. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

## 2. Problem formulation

Consider the following class of discrete time-varying stochastic systems

$$\begin{cases} x(k+1) = \big(A(k) + \sum_{i=1}^{r} A_i(k)w_i(k)\big)x(k) \\ \qquad\qquad + g(k, x(k)) + D(k)v(k) + B_1(k)u(k) \\ z(k) = L(k)x(k) + B_2(k)u(k) \end{cases} \tag{1}$$

and $m$ sensor measurements with randomly occurring saturations

$$\begin{aligned} y_i(k) &= \alpha_i(k)\sigma(C_i(k)x(k)) + (1-\alpha_i(k))C_i(k)x(k) \\ &\quad + E_i(k)\varpi_i(k) \quad (i = 1, 2, \dots, m) \end{aligned} \tag{2}$$

where $x(k) \in \mathbb{R}^{n_x}$ represents the state vector; $y_i(k) \in \mathbb{R}$ is the measurement output measured by sensor $i$ from the plant; $z(k) \in \mathbb{R}^{n_z}$ is the controlled output vector; $u(k) \in \mathbb{R}^{n_u}$ is the control input vector; $w_i(k) \in \mathbb{R}$ $(i = 1, 2, \dots, r)$, $v(k) \in \mathbb{R}^{n_v}$ and $\varpi_i(k) \in \mathbb{R}^{n_w}$ $(i = 1, 2, \dots, m)$ are, respectively, the multiplicative noise, the process noise and the measurement noise for sensor $i$. The noise sequences are mutually uncorrelated zero-mean Gaussian sequences with $\mathbb{E}\{w_i(k)w_i^T(k)\} = 1$, $\mathbb{E}\{v(k)v^T(k)\} = V(k)$ and $\mathbb{E}\{\varpi_i(k)\varpi_i^T(k)\} = \bar{W}_i(k)$. $A(k), A_i(k), B_1(k), B_2(k), D(k), L(k), C_i(k)$ and $E_i(k)$ are known, real, time-varying matrices with appropriate dimensions.

The nonlinear function $g(k, x(k))$ with $g(k, 0) = 0$ is a stochastic nonlinear function having the following statistical characteristics:

$$\mathbb{E}\left\{g(k, x(k))\middle| x(k)\right\} = 0,$$

$$\mathbb{E}\left\{g(k, x(k))g^T(j, x(k))\middle| x(k)\right\} = 0, \quad k \neq j$$