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Brief paper

Iterative learning control for discrete-time systems with event-triggered transmission strategy and quantization[☆]

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ABSTRACT

This paper investigates the iterative learning problem for discrete-time systems with event-triggered scheme and quantization. The event-triggered scheme is firstly considered in the iterative learning controllers to reduce the number of iteration steps to be updated. Here, the event-triggered scheme is designed depending on time t and iterative learning step k. Quantization is then introduced in the event-triggered controllers and some relaxed conditions are presented to guarantee the tracking problem by using some interval matrix properties. Finally, simulation results are given to illustrate the usefulness of the developed criteria.

1. Introduction

Iterative learning control (ILC) is an effective technique that aims to improve the current performance of uncertain systems over a fixed time interval by learning from previous executions (trials, iterations, passes). The focus of ILC is to improve the performance of systems that execute a repeated operation. In the past decades, ILC has been successfully applied to industrial robots (Arimoto, Kawamura, & Miyazaki, 1984), chemical reactors (Mezghani et al., 2002), input saturation (Tan, Xu, Norrlöf, & Freeman, 2011; Xiong, Ho, & Yu, 2015; Xu, Tan, & Lee, 2004; Zhang, Chi, & Ji, 2015a), heat equations (Huang, Xu, Li, Xu, & Yu, 2013), sampled-data systems (Abidi & Xu, 2011), and multi-node systems (Li & Li, 2014; Meng, Jia, & Du, 2015a,b; Meng, Jia, Du, & Yu, 2013; Meng, Jia, Du, & Zhang, 2014; Meng & Moore, 2016). For example, Li et al. in Li and Li (2014) showed that all the followers can track the leader uniformly on the finite interval $[1, T]$ for consensus problem and keep the desired distance from the leader to achieve velocity consensus uniformly on $[1, T]$ for the formation problem. In Meng et al. (2014), Meng et al. dealt with the formation control problems for multi-node systems with nonlinear dynamics and switching network topologies. It was shown in Meng et al. (2015a) that these uncertainties of multi-node systems are dynamically changing not only along the time axis but also along the iteration axis.

In the above literature, the given ILC algorithms are always updated in each iteration step (see Eq. (14) in Tan et al., 2011 and Eq. (4) in Meng et al., 2015a). *However, it is costly and unnecessary to update the ILC algorithm in each iteration step when the iterative controller change little in some successive iteration steps. To reduce the number of iteration steps to be updated, an event-triggered control scheme is introduced in this paper.* In the event-triggered control, the measurement error plays a key role in the event design. When the measurement error reaches the prescribed threshold, an event is triggered and the controller is updated. In the recent years, as a good digital control scheme that can be used to reduce communication load, event-triggered control has been receiving increasing attention in wireless sensor/actuator systems (Mazo & Tabuada, 2011), networked control systems (Yue, Tian, & Han, 2013), fuzzy systems (Peng, Han, & Yue, 2013), sampled-data control systems (Peng & Han, 2013; Zou, Wang, Gao, & Liu, 2015), multi-node systems (Fan, Feng, Wang, & Song, 2013; Hu, Liu, & Feng, 2015; Zhang, Hao, Zhang, & Wang, 2014). *Different with the existing literature, the event-triggered scheme will be applied on ILC algorithm in this paper and the event-triggered iterative learning*

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controller will be designed to be related to time t and iterative learning k.

In addition, in some applications, such as sensor systems and industrial control systems, the aim is to control multiple dynamical systems by using multiple sensors to exchange information over a communication network. Because of the limitation of storage and communication bandwidth among nodes, the original precise information needs to be quantized. Hence, it is necessary to conduct analysis on the quantizers and understand how much effect the quantization makes on dynamic systems. In fact, the problem of quantized control for dynamic systems has been available in the literature (Delchamps, 1990; Fu & Xie, 2005; Liu, Guan, Li, Zhang, & Xiao, 2012; Wang, Shen, Shu, & Wei, 2012; Zhang, Zhang, Hao, & Wang, 2015b). Unfortunately, to the best of our knowledge, *quantized ILC problem for dynamic systems has not been fully investigated despite its potential in practical applications. Furthermore, event-driven mechanism has not been used in ILC to reduce the requirements of storage and communication bandwidths. The aim of this paper is to address these problems.*

Hence, our objectives in this study are twofold: (1) Consider the event-triggered scheme in the iterative learning controllers to reduce the number of iteration steps to be updated and discuss the tracking problem of discrete-time systems with event-triggered scheme in a finite interval; (2) Consider quantization in event-triggered controllers for discrete-time systems, and present some relaxed conditions to solve the tracking problem of the discussed systems by using some interval matrix properties.

The remainder of this paper is organized as follows: The problem formulation is presented in Section 2. In Section 3, the event-triggered scheme is considered in the iterative learning controllers. Moreover, quantization is applied in the event-triggered controllers. In Section 4, simulations are carried out to illustrate the effectiveness of the main results. Finally, conclusions are drawn in Section 5.

Notation: Throughout this study, the superscript T represents the transpose. For all $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$, define $\|x\| = \left(\sum_{i=1}^n x_i^2\right)^{\frac{1}{2}}$. For a matrix A, $\|A\|$ denotes the spectral norm defined by $\|A\| = (\lambda_M(A^TA))^{\frac{1}{2}}$, and $\rho(A)$ is the spectral radius with $\rho(A) = \max_i |\lambda_i(A)|$, where $\lambda_i(A)$ denotes the ith eigenvalue of matrix A, respectively.

2. Preliminaries

Consider an iterative learning system consisting of N nodes (N is a positive integer). Each node has to deal with two independent dynamic processes: the first process shows the system dynamics about time t; the second process describes the system dynamics of node i about iterative learning k. Hence, the system dynamics are described by

$$x_i(t+1,k) = c_i x_i(t,k) + b_i u_i(t,k), \tag{1}$$

where $x_i(t,k)$ is the state vector of node i, $i = 1, 2, \ldots, N$, and $x(t,k) = (x_1(t,k), x_2(t,k), \ldots, x_N(t,k))^T \in R^N$; $t \in \{0, 1, \ldots, T\}$ ($T > 0$ is a positive integer) and $k \in Z_+$ (Z_+ is the set of nonnegative integers); $C = diag(c_1, c_2, \ldots, c_N) \in R^{N\times N}$ and $B = diag(b_1, b_2, \ldots, b_N) \in R^{N\times N}$ are constant matrices; $u_i(t,k)$ is the iterative learning controller of node i and $u(t,k) = (u_1(t,k), u_2(t,k), \ldots, u_N(t,k))^T \in R^N$. For every node i ($i \in \{1, 2, \ldots, N\}$) in system (1), it is said to achieve the tracking of a desired reference trajectory if

$$\lim_{k\to+\infty} x_i(t,k) = x^*(t), \quad t \in \{0, 1, \ldots, T\}, \tag{2}$$

where $x^*(t) \in R$ is the desired reference trajectory. Let $e_i(t,k) = x_i(t,k) - x^*(t)$ be the tracking error of node i, and $e(t,k) = (e_1(t,k),\ e_2(t,k), \ldots, e_N(t,k))^T$. Note that the tracking objective (2) holds if and only if $\lim_{k\to+\infty} e(t,k) = 0$, for $\forall\, t \in \{0, 1, \ldots, T\}$.

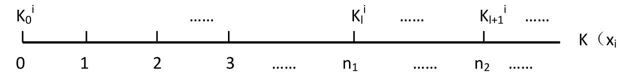

Fig. 1. The broadcasting iteration sequence $\{k_l^i\}$ of node i.

In the existing literature (see Meng et al., 2015a, 2014), the ILC $u(t,k)$ is always updated in each iteration k. However, it is not necessary to update the ILCs if the changes of controllers in some successive iteration steps are small. Hence, the ILCs with event-triggered strategy will be considered in this paper. We define the state measurement error of node i by $\delta_i(x_i(t,k)) = \widehat{x}_i(t,k) - x_i(t,k)$, $\forall\ i \in \{1, 2, \ldots, N\}$, $t \in \{0, 1, \ldots, T\}$, and $\forall\ k \in Z_+$. Here, $\widehat{x}_i(t,k)$ denotes the latest sampled state of node i, which will be given later. Note that, the transmission information needs to be coded or quantized due to the limitation of storage and communication bandwidth among nodes. Hence, based on the measurement error of node i, the event-triggered strategies without and with quantization are given in the following

$$|\delta_i(x_i(t,k))| = \gamma_i \cdot \left|\sum_{j=1}^n l_{ij}\widehat{x}_j(t,k)\right|, \tag{3}$$

$$|\delta_i(x_i(t,k))| = \gamma_i \cdot \left|\sum_{j=1}^n l_{ij} q(\widehat{x}_i(t,k))\right|, \tag{4}$$

where $\gamma_i > 0$, matrix $L = (l_{ij})_{N\times N} \in R^{N\times N}$, and $\|L\| \neq 0$. And $q(\cdot) : R \to \Lambda_\varpi$ is a logarithmic quantizer, for a given accuracy parameter $\varpi \in (0,1)$, one can define the logarithmic set of quantization levels

$$\Lambda_\varpi = \{\pm\omega_{(i)} : \omega_{(i)} = \varpi^i \omega_{(0)},\ i = \pm1, \pm2, \ldots\} \bigcup \{\pm\omega_{(0)}\} \bigcup \{0\}, \quad \omega_{(0)} > 0. \tag{5}$$

The associated quantizer $q(\cdot)$ is defined as follows:

$$q(x) = \begin{cases} \omega_{(i)}, & \text{if } \dfrac{1}{1+\sigma}\omega_{(i)} < x \le \dfrac{1}{1-\sigma}\omega_{(i)}; \\ 0, & \text{if } x = 0; \\ -q(-x), & \text{if } x < 0, \end{cases} \tag{6}$$

where $\sigma = \frac{1-\varpi}{1+\varpi}$ is named sector bound in Fu and Xie (2005). If event triggering condition (3) (or (4)) is satisfied, node i will broadcast its state and update its control protocol. The information broadcasting iteration sequence of node i is $\{k_l^i\}$ ($i \in \{1, 2, \ldots, N\}$, $l \in Z_+$) (see Fig. 1). And $\widehat{x}_i(t,k)$ is defined as $\widehat{x}_i(t,k) = x_i(t,k_l^i)$, $k \in [k_l^i, k_{l+1}^i)$, $l \in Z_+$. From the definition of logarithmic quantizer, the quantization error satisfies the following condition:

$$q(\widehat{x}_i(t,k)) = \widehat{x}_i(t,k) + \widetilde{\Lambda}(t,k)\widehat{x}_i(t,k), \tag{7}$$

where $\widetilde{\Lambda}(t,k)$ is a scalar and satisfies that $\widetilde{\Lambda}(t,k) \in [-\sigma, \sigma]$, $\forall\, t \in \{0, 1, \ldots, T\}$, $i \in \{1, 2, \ldots, N\}$ and $\forall\, k \in Z_+$.

According to the event triggering conditions (3) and (4), we shall consider the ILCs with and without quantization, which can be expressed as

$$\widehat{u}_i(t, k_{l+1}^i) = \widehat{u}_i(t, k_l^i) + \Gamma_1^i \sum_{j=1}^n l_{ij}\widehat{x}_i(t+1,k), \tag{8}$$

$$\widehat{u}_i(t, k_{l+1}^i) = \widehat{u}_i(t, k_l^i) + \Gamma_2^i \sum_{j=1}^n l_{ij} q(\widehat{x}_i(t+1,k)), \tag{9}$$

and $u_i(t,k) = \widehat{u}_i(t,k_l^i)$, $k \in [k_l^i, k_{l+1}^i)$, $l \in Z_+$, $\Gamma_1 = diag(\Gamma_1^1, \Gamma_1^2, \ldots, \Gamma_1^N)$ and $\Gamma_2 = diag(\Gamma_2^1, \Gamma_2^2, \ldots, \Gamma_2^N)$ are learning gain matrices, which will be designed later.