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Brief paper

Sampled-data-based stabilization of switched linear neutral systems ☆

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ABSTRACT

With the development of digital control technology, sampled-data control shows its prominent superiority for most practical industries. In the framework of sampled-data control, this paper studies the stabilization problem for a class of switched linear neutral systems meanwhile taking into account asynchronous switching. By utilizing the relationship between the sampling period and the dwell time of switched neutral systems, a bond between the sampling period and the *average* dwell time is revealed to form a switching condition, under which and certain control gains conditions exponential stability of the closed-loop systems is guaranteed. A simple example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

A switched system consists of a family of continuous-time or discrete-time subsystems and a switching law to determine which subsystem is active within certain time interval. Lots of works are devoted to switched systems in the past two decades, see, for example Branicky (1998), Chen and Zheng (2010), Fu, Ma, and Chai (2015), Lian, Ge, and Han (2013), Liberzon (2003), Lin and Antsaklis (2009), Sun, Du, Shi, Wang, and Wang (2014), Sun and Ge (2005), Sun, Liu, David, and Wang (2008), Sun, Zhao, and Hill (2006), Wu and Dong (2006), Xiang, Sun, and Chen (2012), Zhai, Hu, Yasuda, and Michel (2001), Zhang and Gao (2010), Zhang and Yu (2009), Zhang, Zhuang, and Shi (2015), Zhang, Zhuang, Shi, and Zhu (2015), Zhao and Hill (2008) and Zhao, Shi, and Zhang (2012). Among all problems studied for switched systems, asynchronous switching stemming from the delay between the active subsystem and its matched controller is one important issue (Lian et al., 2013; Wang, Zhao, & Jiang, 2013; Xiang et al., 2012; Zhang & Gao, 2010; Zhao et al., 2012). A majority of existing literature on asynchronous switching focus on either for continuous-time switched systems with continuous controllers or for discrete-time switched systems with discrete-time controllers. The authors of Zhang and Gao (2010) study asynchronously switching control of switched systems with average dwell time technique in both continuous-time and discrete-time cases, where the asynchronous delay is not necessarily less than the dwell time. Refs. Lian et al. (2013) and Zhao et al. (2012) introduce asynchronously switching control methods for deterministic switched linear systems and stochastic ones, respectively. Both methods also deploy the average dwell time technique. Since most practical systems are continuous-time, for which one usually either directly designs continuous-time controllers or first discretizes the continuous systems and then develops the corresponding discrete-time controllers, for example, those methods in Lian et al. (2013), Wang et al. (2013) and Zhang and Gao (2010). However, from a practical implementation point of view, sampled-data controllers are more favorable in practical applications due to the rapid progress of computer and digital technologies and non-approximate treatment of the intersample behavior from sampled-data control's own features, which results in no degradation of the closed-loop performance (Chen & Francis, 1991, 1995; Hara, Yamamoto, & Fujioka, 1996; Hu, Lam, Cao, & Shao, 2003). Given the advantages of sampled-data control, this paper considers a particular class of switched systems called switched neutral system (Krishnasamy & Balasubramaniam, 2015a; Liu, Liu, & Zhong, 2008; Li, Zhao, & Qi, 2014; Wang et al., 2013; Xiang, Sun, & Chen, 2011; Xiang, Sun, & Mahmoud, 2012;

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Xiong, Zhong, Ye, & Wu, 2009; Zhang, Liu, Zhu, & Zhong, 2007; Zhang & Yu, 2012) which is first defined in Sun, Fu, Sun, and Zhao (2005), and study its stabilization *in the framework of sampled-data control* meanwhile taking into account asynchronous switching.

To this considered problem, the most relevant works are Feng and Song (2011), Krishnasamy and Balasubramaniam (2015b), Liberzon (2014), Lien, Chen, Yu, and Chung (2012) and Wang, Xing, Zhou, Wang, and Yang (2014). The authors of Wang et al. (2014) propose a finite-time stabilizer for a class of switched linear systems under asynchronous switching. The authors of Liberzon (2014) present an important result on sampled-data quantized state feedback stabilization of switched linear systems by using an encoding and control strategy. In Feng and Song (2011), the authors investigate the stabilization problem of switched linear systems for both the known switching process and the unknown switching setting, for which cases the dwell time technique and the online adaptive estimation method are combined with the sampled-data control, respectively. In Feng and Song (2011), Liberzon (2014) and Wang et al. (2014), only the delay-free switched linear systems are considered. In Lien et al. (2012), the authors study the robust delay-dependent H_∞ control for a class of switched linear delay systems. However, the authors convert the sampled-data control problem into a time-delay one and used time-varying delay approach instead of sampled-data feedback control input. Furthermore, impacts from sampling or from asynchronous switching are not considered on stabilization of switched systems. Although all these references give sampled-data control strategies for switched systems with their own spectacular features, they are obviously not able to cope with stabilization of switched *neutral* systems under either asynchronous or synchronous switching because time delays appear not only in states but also in state derivative of the considered system (see Eq. (1)). In Krishnasamy and Balasubramaniam (2015b), the authors study the sampled-data control for switched neutral systems under synchronous switching which possesses great extent conservativeness. Thus one may wonder whether or not it is possible to propose a new control stabilization method for switched neutral systems, even for switched linear ones, in the framework of sampled-data control under asynchronous switching? This paper provides an affirmative answer.

From the motivation above, this paper focuses on a class of switched linear neutral systems and introduces a sampled-data-based controller for *asynchronously* switching stabilization. By utilizing the relationship between the sampling period and the dwell time of switched systems, a bond between the sampling period and the *average* dwell time is revealed to form a switching condition. Subject to this switching condition and certain control gains related constraints, exponential stability of the closed-loop switched neutral system can be guaranteed. The main features of this paper are as follows. (1) We, for the first time, propose a sample-data-based stabilization method for switched linear neutral systems under asynchronous switching. (2) Delays in the switched neutral systems are time-varying and also appear in the state derivative. (3) A bond between the sampling period and the *average* dwell time is revealed to form a switching condition. (4) Introducing free-weighting matrices obtains the decoupled constraints (30) and (31), and therefore technically reduces the computational complexity compared to the result of Wang et al. (2013).

This paper is organized as follows. Section 2 describes the problem statement and gives some useful definitions and lemmas. Section 3 gives the controller design of stabilizing the switched neutral system under sampled-data input. An example is presented in Section 4, and followed by the conclusion in Section 5.

Notations: $C_r = C([-r, 0], R^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-r, 0]$ into R^n with the topology of uniform convergence. $\|\varphi\|_C = \sup_{-r\le t\le 0}\|\varphi(t)\|$ denotes the norm of a function $\varphi \in C_r$. R^n is the n-dimensional Euclidean space. $Q > 0 (Q < 0)$ means that the matrix Q is positive definite (negative definite). Q^T and Q^{-1} present the transpose and the inverse of the matrix Q, respectively. $\|\cdot\|$ is the Euclidean norm. $*$ symbolizes the elements below the main diagonal of a symmetric matrix. $\underline{\lambda}(Q)$ and $\bar{\lambda}(Q)$ are the smallest and the largest eigenvalue of a matrix Q, respectively. $\mathcal{N}$ is the set of nonnegative integers. diag$\{\cdots\}$ denotes a block-diagonal matrix.

2. Problem statement

Consider the switched neutral system

$$\begin{cases}\dot{x}(t) = A_\sigma x(t) + B_\sigma x(t-\tau(t)) + C_\sigma \dot{x}(t-h(t)) + D_\sigma u(t)\\ x(t_0+\theta) = \varphi(\theta), \quad \theta\in[-r,0]\end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control vector, $\sigma : [0, \infty) \to \mathcal{M} = \{1, 2, \ldots, m\}$ is a right-continuous, piecewise constant function called the switching signal, $\{(A_i, B_i, C_i, D_i) : i \in \mathcal{M}\}$ is a collection of matrix pairs defining the individual subsystem of the system (1), and all the eigenvalues of matrix C_i are inside the unit circle, $\tau(t)$ and $h(t)$ denote the discrete time-varying delay and the neutral time-varying delay, respectively, which satisfy

$$\begin{aligned} &0 < \tau(t) \le \tau, \qquad \dot{\tau}(t) \le \hat{\tau} < 1,\\ &0 < h(t) \le h, \qquad \dot{h}(t) \le \hat{h} < 1, \end{aligned} \tag{2}$$

where $\tau, \hat{\tau}, h$ and $\hat{h}$ are constants. $\varphi(\theta)$ is a continuously differential vector initial function on $[-r, 0]$, $r = \max\{\tau, h\}$. We denote the number of discontinuities of the switching signal σ on the interval $(s, t]$ by $N_\sigma(t, s)$. For all $i \in \mathcal{M}$, we assume that the subsystem i is stabilizable and there is no jump at all switching instants.

For the purpose of this paper, the definitions of average dwell time and exponential stability of the switched linear neutral system are introduced below.

Definition 1 (*Liberzon, 2003*)**.** If there exists a number $\tau_d > 0$ such that any two switches are separated by at least τ_d, then τ_d is called the dwell time. In addition, if there exist two positive numbers $\tau_a > \tau_d$ and $N_0 \ge 1$ such that

$$N_\sigma(t, s) \le N_0 + \frac{t-s}{\tau_a} \quad \forall t \ge s \ge 0, \tag{3}$$

then τ_a is called the average dwell time.

Definition 2 (*Wang et al., 2013*)**.** The system (1) is said to be exponentially stable under a switching law σ, if the solution $x(t)$ of the system (1) satisfies

$$\|x(t)\| \le \kappa e^{-\lambda(t-t_0)}\|x_{t_0}\|_c, \quad t \ge t_0 \tag{4}$$

for constants $\kappa \ge 1$ and $\lambda > 0$, where $\|x_{t_0}\|_c = \sup_{-r\le\theta\le 0}\{\|x(t_0+\theta)\|, \|\dot{x}(t_0+\theta)\|\}$.

State measurements are taken at times $t_k := k\tau_s, k \in \mathcal{N}$, where τ_s is a fixed sampling period. When the sampler gets the state information, it also passes the switching signal into the controller. However, the sampling time and the switching time do not happen synchronously. The delayed time will last to the next sampling time. During this period, the active subsystem would be unstable. Thus the control objective is to stabilize the system (1) by designing a sampled-data controller meanwhile taking into account asynchronous switchings. In order to simplify the analysis process, we assume the sampling period τ_s is no larger than the dwell time τ_d.