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Brief paper

Finite-horizon Gaussianity-preserving event-based sensor scheduling in Kalman filter applications☆

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ABSTRACT

This paper considers a remote state estimation problem, where a sensor measures the state of a linear discrete-time system. The sensor has computational capability to implement a local Kalman filter. The sensor-to-estimator communications are scheduled intentionally over a finite time horizon to obtain a desirable tradeoff between the state estimation quality and the limited communication resources. Compared with the literature, we adopt a Gaussianity-preserving event-based sensor schedule bypassing the nonlinearity problem met in threshold event-based polices. We derive the closed-form of minimum mean-square error (MMSE) estimator and show that, if communication is triggered, the estimator cannot do better than the local Kalman filter, otherwise, the associated error covariance, is simply a sum of the estimation error of the local Kalman filter and the performance loss due to the absence of communication. We further design the scheduler's parameters by solving a dynamic programming (DP) problem. The computational overhead of the DP problem is less sensitive to the system dimension compared with that of existing algorithms in the literature.

1. Introduction

The concept of controlled communication for the state estimation of a dynamical system has been prevailing in recent years. Controlled communication in general refers to reducing the communication rate intentionally to obtain a desirable tradeoff between the state estimation quality and the limited communication resources. This is rooted in the fact that the communication between the wireless sensors and the estimator at full rate is unlikely to occur for most practical applications. For instance, since the sensors are usually battery-powered and sparsely deployed, the replacement of onboard battery is not possible in most occasions. Reducing the communication rate is reasonably an alternative approach to resolve the energy saving problem. Another incentive for controlled communication is to avoid traffic congestion of the network shared by a vast number of sensors.

Estimation error covariance is most widely used for measuring the estimation quality. To minimize inevitable enlarged estimation error covariance due to the reduced communication rate, a communication scheduling strategy for a sensor is needed. Yang and Shi (2011) provided an insight that communications should be initiated periodically or more generally, as uniformly as possible, to minimize the average error covariance. For the so-called variance-based triggered scheduling in Trimpe and D'Andrea (2014), covariance recursion asymptotically converges to a periodic one. Informally, purely using the information in the error covariance is likely to lead to a periodic communication schedule. Another line of research direction such as Han, Cheng, Chen, and Shi (2013), Shi, Chen, and Darouach (2016), Shi, Chen, and Shi (2015) and Shi, Elliott, and Chen (2016) is the event-based sensor scheduling, where communication is triggered by a certain event defined on the system state. Threshold event-based communication schedules have been proposed by Battistelli, Benavoli, and Chisci (2012), Lipsa and Martins (2011), Molin (2014), Wu, Jia, Johansson, and Shi (2013) and Xu and Hespanha (2005), in different contexts but can hardly generate closed-form of the minimum mean-square error (MMSE) estimates. To obtain a tractable and simple

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estimator, Han et al. (2015) proposed a stochastic event-based mechanism, bypassing the nonlinear problem met in threshold event-based policies.

In this work we focus on a finite-horizon sensor communication scheduling problem. The sensor, as a smart one, has computational capability to implement a local Kalman filter. The utilization of the onboard computation unit has been shown to help improve estimation performance (Hovareshti, Gupta, & Baras, 2007). To alleviate the degradation of estimation performance, we adopt an event-based sensor scheduling mechanism. The benefit we obtain from this type of mechanisms is attributed to the fact that the absence of triggering provides side information to the estimator. If we pursue an optimal event-based law, it is very likely that the Gaussianity of the conditional distribution in the system state will be destroyed, for which no closed-form expression of the MMSE estimate can be derived. The distribution propagation turns out to be computationally costly under non-Gaussian circumstances. In summary, the information contained in the absence of triggering, on one hand, mitigates the Kalman filtering's performance degradation, but on the other hand, may cause difficulty in computing distribution propagation. To tackle the challenge, in this paper we introduce a similar stochastic event-based mechanism used in Han et al. (2015). Compared with Han et al. (2015), the contribution of this paper is summarized as follows:

(1) We use a simple static parameter estimation example to motivate the stochastic event-based scheduling policy. In the example, the stochastic strategy maintains the *a posteriori* distributions Gaussian with possibly the least variance.
(2) We present a closed-form expression of the MMSE estimate for the remote estimator and show that, if and when the communication is triggered, the estimator cannot do any better than the local Kalman filter, otherwise, the associated error covariance, is simply a sum of the estimation error of the local Kalman filter and the performance loss due to the absence of communication.
(3) The sensor scheduling problem can be modeled as a decision process. The sensor can sequentially design the scheduler's parameters by solving a dynamic programming (DP) problem, efficiently allocating communication resource over a finite time-horizon. The computational overhead of the DP problem is less sensitive to the dimension of systems compared with the existing works.

Notation: $\mathbb{N}$ is the set of positive integers numbers. $\mathbb{S}_+^n$ is the set of n by n symmetric positive semi-definite matrices over the real field. The notation $p(\mathbf{x}, x)$ represents the probability density function (pdf) of a random variable $\mathbf{x}$ taking value at x. For a matrix X, we abuse the notations $\det(X)$ and X^{-1}, in case of a singular matrix X, to respectively denote the pseudo-determinant and the Moore–Penrose pseudoinverse of X. The notation $X^{1/2}$ is the square root of a positive semidefinite matrix X. For a Borel set $\mathcal{B}$, $\mathfrak{L}(\mathcal{B})$ stands for the Lebesgue measure. $\times$ denotes Cartesian product and $\oplus$ stands for Minkowski addition of two sets, respectively. Define the function h: $\mathbb{S}_+^n \mapsto \mathbb{S}_+^n$ as $h(X) \triangleq AXA' + Q$.

2. Kalman filter under controlled communication

Consider a linear time-invariant system:

$$x_{k+1} = Ax_k + w_k, \tag{1a}$$

$$y_k = Cx_k + v_k, \tag{1b}$$

where $x_k \in \mathbb{R}^n$ is the system state vector and $y_k \in \mathbb{R}^m$ is the observation vector. The noises $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are zero-mean Gaussian random vectors with $\mathbb{E}[w_k w_j'] = \delta_{kj} Q$ ($Q \geq 0$), $\mathbb{E}[v_k v_j'] = \delta_{kj} R$ ($R > 0$), where δ_{kj} is the Kronecker delta function with $\delta_{kj} = 1$ if $k = j$ and 0 otherwise, and $\mathbb{E}[w_k v_j'] = 0$ $\forall j, k$. The initial state x_0 is a zero-mean Gaussian random vector that is uncorrelated with w_k and v_k and has covariance $\Sigma_0 \geq 0$. The pair (C, A) is assumed to be observable and $(A, Q^{1/2})$ is controllable.

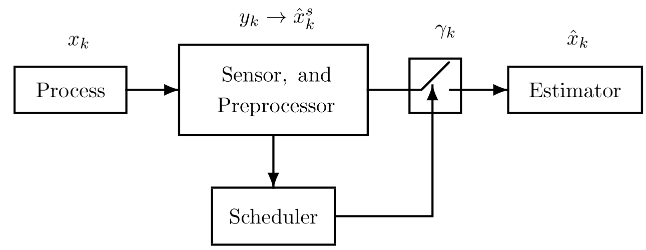

Fig. 1. Remote state estimation with a communication scheduler.

All the measurements collected by the sensor up to time k is denoted by $y_{1:k} \triangleq \{y_1, \dots, y_k\}$. The sensor locally computes $\hat{x}_k^s \triangleq \mathbb{E}[x_k | y_{1:k}]$, the MMSE estimate of x_k based on $y_{1:k}$. Let P_k^s be the associated estimation error covariance matrix, i.e., $P_k^s \triangleq \mathbb{E}[(x_k - \hat{x}_k^s)(x_k - \hat{x}_k^s)' | y_{1:k}]$, which is computed via a standard Kalman filter initialized with $\hat{x}_0^s = 0$ and $P_0^s = \Sigma_0$. The sensor is equipped with a transmission scheduler (see Fig. 1), which determines whether or not $\hat{x}_k^s$ should be sent to the estimator, according to the history of transmission decision actions and the measurements collected by the sensor up to time k. Let $\gamma_k \in \{0, 1\}$ denotes the communication decision made by the scheduler. If $\gamma_k = 1$, $\hat{x}_k^s$ is sent; otherwise $\hat{x}_k^s$ is not sent. Since the sensor local estimation is initialized with $\hat{x}_k^s = 0$, without loss of generality, we assume $\gamma_0 = 1$. To focus on the role of the sensor scheduler in achieving a desired tradeoff between the remote estimation quality and communication resource, other aspects of imperfect communication, such as packet dropouts, delays and data quantization, will not be taken into account, that is, if sent by the senor, the data will reach the estimator side.

It should be noted that before deciding γ_k at time k, $\gamma_{1:k-1} \triangleq \{\gamma_1, \dots, \gamma_{k-1}\}$ is known by the sensor. Besides, the sensor has all the measurements collected by itself. The information pattern of the sensor up to after communication at time k, if any, is denoted as $\mathcal{I}_k^{\mathrm{S}}$, i.e.,

$$\mathcal{I}_k^{\mathrm{S}} \triangleq \{y_1, \dots, y_k\} \cup \{\gamma_1, \dots, \gamma_k\}, \quad \text{with } \mathcal{I}_0^{\mathrm{S}} = \emptyset.$$

Similarly, we denote by $\mathcal{I}_k^{\mathrm{E}}$ the information pattern at the remote estimator up to after communication at time k. Because of the perfect communication channel assumed, γ_k is known to the estimator. $\mathcal{I}_k^{\mathrm{E}}$ contains both the history of communication actions $\gamma_{1:k}$ and the measurement data received from the sensor, that is,

$$\mathcal{I}_k^{\mathrm{E}} = \{\gamma_1 \hat{x}_1^s, \dots, \gamma_k \hat{x}_k^s\} \cup \{\gamma_1, \dots, \gamma_k\}, \quad \text{with } \mathcal{I}_0^{\mathrm{E}} = \emptyset.$$

We define a communication scheduling policy applied by the sensor at time k as a function f_k:

$$\gamma_k = f_k(\mathcal{I}_{k-1}^{\mathrm{S}}, y_k), \tag{2}$$

where f_k's are assumed to be measurable mappings. A finite-horizon sensor communication policy Θ is accordingly defined as a sequence of f_k's: $\Theta \triangleq \{f_1, f_2, \dots, f_T\}$. Because the estimator is aware of Θ being used by the sensor, it computes $\tilde{x}_k$, its own estimate of the state x_k based on $\mathcal{I}_k^{\mathrm{E}}$, $\tilde{x}_k = g_k(\mathcal{I}_k^{\mathrm{E}})$, where g_k's are measurable mappings. A finite-horizon remote estimator Ξ is accordingly defined as a sequence of g_k's: $\Xi \triangleq \{g_1, g_2, \dots, g_T\}$. The estimator computes P_k, the corresponding estimation error covariance matrix, as: $P_k = \mathbb{E}_\Theta\left[(x_k - \tilde{x}_k)(x_k - \tilde{x}_k)' \mid \mathcal{I}_k^{\mathrm{E}}\right]$, where $\mathbb{E}_\Theta[\cdot|\cdot]$ denotes conditional expectation with respect to a fixed Θ.