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Brief paper

Mean square state estimation for sensor networks[☆]

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ABSTRACT

This paper deals with mean square state estimation over sensor networks with a fixed topology. Attention is focused on designing local stationary state estimators with a general structure while accounting for the network communication topology. Two estimator design approaches are proposed. One is based on the observability Gramian, and the other on the controllability Gramian. The computation of the estimator state-space matrices is recast as off-line convex optimization problems and requires the system asymptotic stability and global knowledge of the network topology. Convergence of the estimation error variance is ensured at each network node and a guaranteed performance in the mean square sense is achieved. The proposed approaches are also extended for designing robust filters to handle polytopic-type parameter uncertainty.

1. Introduction

A wide range of systems are monitored by sensor networks. This has resulted in an important research activity on decentralized state estimation for linear dynamic systems, as surveyed in Farina, Espinosa, Garcia, and Scattolini (2011). The work reported here deals with sensor networks with neighbor-to-neighbor communication. Specifically, at a given node, information is only received from a subset of nodes with which it can communicate directly. The considered decentralized state estimation problem aims at estimating the whole state vector at each sensor node on the basis of the overall model of the system (Battistelli & Chisci, 2014; Cattivelli & Sayed, 2010; Olfati-Saber, 2007, 2009; Ugrinovskii, 2011). This problem is referred to as distributed state estimation. It should be distinguished from partition-based state estimation where only a local model of the observed system is available at each node and the problem amounts to estimate the part of the system state associated to this local model at each node (Farina, Ferrari-Trecate, & Scattolini, 2010; Khan & Moura, 2008; Stankovic, Stankovic, & Stipanovic, 2009a,b).

Distributed state estimation was addressed initially by including an additional step to the local Kalman filter, besides the measurement update step and the prediction step, a so-called consensus or diffusion step. This new step consists in propagating the state estimate (Cattivelli & Sayed, 2010; Olfati-Saber, 2009) or the whole probability density function between neighboring nodes (Battistelli & Chisci, 2014).

The properties of the different schemes were analyzed in terms of stability and performance. For most existing algorithms, some sort of local observability or detectability is required to ensure stability. Exceptions include Battistelli and Chisci (2014) and Ugrinovskii (2011), which only require collective detectability. As far as performance is concerned, the different distributed observer schemes are often compared through case studies with the centralized Kalman filter, which is seen as a benchmark. However, the selection of the observer gains to meet some optimality criterion is only considered in a few studies. Different criteria have been proposed. In Alriksson and Rantzer (2006) the minimization of the mean square estimation error at each node is considered. Other problem formulations adopt the minimization of the mean square estimation error over the whole network (Carli, Chiuso, Schenato, & Zampieri, 2008; Stankovic et al., 2009a), the minimization of the H_∞ gain from noise to estimation error (Li &

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Sanfelice, 2014), or the minimization of the worst case consensus performance (Ugrinovskii, 2011). In the case of mean square error minimization, only approximate solutions were reported. Indeed, either the classical Kalman filter recurrence is used in the measurement update step, or the consensus gains are restricted to a certain structure or even imposed *a priori*. On the other hand, in Ugrinovskii (2011), the minimization of the worst case consensus is achieved through an LMI-based design method. However, this criterion tends to enforce high interconnection (or consensus) gains, which can be alleviated by introducing additional convex constraints in order to limit the magnitude of these gains.

For handling modeling uncertainties, an adaptive sliding mode observer has been considered in Menon and Edwards (2013) to cope with norm-bounded uncertainties, whereas an LMI-based approach to achieve robustness with respect to parametric uncertainties meeting a matching condition, as well as sector nonlinearities, has been proposed in Lv, Liang, and Cao (2011). As for distributed observers for nonlinear systems, recent contributions include Battistelli, Chisci, Mugnai, and Farina (2015), Farina, Ferrari-Trecate, and Scattolini (2012), and Zeng and Liu (2015).

This paper aims at designing stationary local state estimators with an optimized mean square state estimation error. A certain degree of consensus between the different nodes is enforced by propagating the estimator state vector between neighboring nodes. The main contribution of this paper is the development of LMI-based methods for designing local state estimators with a general structure. The methodology is first developed for uncertainty-free linear time-invariant systems. Then, it is extended to deal with polytopic-type parameter uncertainties. The proposed approach applies only to asymptotically stable systems, with the filters design being carried out off-line and requires global knowledge of the network topology.

Notation. $\mathscr{N}$ is the set $\{1, 2, \dots, n\}$, $\mathbb{Z}^+$ is the set of nonnegative integers, $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices, I_n is the $n \times n$ identity matrix, and $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix. For a matrix S, S' denotes its transpose, $S > 0$ ($S \geq 0$) means that S is symmetric and positive-definite (positive semi-definite), $S^{-T} := (S^{-1})'$, and $\mathscr{E}\{\cdot\}$ denotes mathematical expectation.

2. Problem statement

Consider a dynamic system with the state-space model

$$x(k+1) = Ax(k) + Bw(k), \quad x(0) = x_0, \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state, $w(k) \in \mathbb{R}^{n_w}$ is the process noise, and A and B are constant matrices with appropriate dimensions. Associated to this system, there exists a sensor network consisting of n sensing nodes given by:

$$y_i(k) = C_i x(k) + D_i v_i(k), \quad i = 1, \dots, n, \tag{2}$$

where $y_i(k) \in \mathbb{R}^{n_i}$ is the ith measurement, $v_i(k) \in \mathbb{R}^{m_i}$ is the ith measurement noise, and C_i and D_i are matrices with appropriate dimensions. The communication topology of the sensor network is defined by the adjacency matrix as follows:

$$\Gamma = [\gamma_{ij}], \quad i, j = 1, \dots, n, \tag{3}$$

where γ_{ij} is a known binary variable indicating whether information is sent from node j to node i (when $\gamma_{ij} = 1$) or not (when $\gamma_{ij} = 0$). Also, let $\gamma_{ii} = 1$. Moreover, let the set $\mathscr{N}_i$ of the neighboring nodes of node i, for all $i \in \mathscr{N}$, be as below:

$$\mathscr{N}_i := \{ j \in \mathscr{N} : \gamma_{ij} \neq 0 \}, \quad i \in \mathscr{N}. \tag{4}$$

It is assumed that $w(k)$ and $v_i(l)$, $i = 1, \dots, n$, for any k, $l \in \mathbb{Z}^+$, are uncorrelated zero-mean white sequences with covariance matrices W and V_i, $i = 1, \dots, n$, respectively, and that x_0 is a random variable which is uncorrelated with $w(k)$ and $v_i(k)$, $i = 1, \dots, n$, for any $k \in \mathbb{Z}^+$.

In a centralized state estimation framework, namely when all measurements are processed together, the minimum variance state estimator for system (1) corresponds to the centralized Kalman filter (CKF). This paper is aimed at designing local state estimators that achieve a performance as close as possible to the CKF. Attention is focused on designing stationary local filters with guaranteed convergence and performance. It is assumed that the sensor nodes may receive the measurement and the state of the estimator of its neighboring nodes. The structure of the filter at each node is quite general, in the sense of not necessarily including a copy of the system dynamics. Specifically, the following network of state estimators is considered:

$$\begin{cases} \xi_i(k+1) = \displaystyle\sum_{j\in\mathscr{N}_i} F_{ij}\xi_j(k) + G_{ij}y_j(k), \quad \xi_i(0) = 0, \\ \qquad\qquad i = 1, \dots, n, \\ \hat{x}_i(k) = \displaystyle\sum_{j\in\mathscr{N}_i} H_{ij}\xi_j(k), \quad i = 1, \dots, n, \end{cases} \tag{5}$$

where $\xi_i(k) \in \mathbb{R}^{n_x}$ is the state of the estimator at node i, $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ is the estimate of $x(k)$ at node i, and F_{ij}, G_{ij} and H_{ij} are matrices of appropriate dimensions to be designed for all $i \in \mathscr{N}$ and $j \in \mathscr{N}_i$.

Note that the estimator at each node, in addition to its own measurement, also incorporates both the measurements and the estimator state vectors of its neighboring nodes. However, when due to communication constraints it is not possible to transmit information from some of the neighboring nodes (either estimator states, measurements or both), the corresponding matrices F_{ij}, G_{ij} and H_{ij} for these nodes are set to zero. In contrast with most of the distributed estimators proposed in the literature, the local estimators in (5) are not constrained to have an observer-type structure, i.e., the system dynamics plus a correction gain times the output estimation error. These features allow for achieving improved performance as will be illustrated by an example in Section 6. Observe that distributed stationary observer-type state estimators, including those using consensus and diffusion strategies are special cases of (5) for particular choices of $F_{i,j}$, $G_{i,j}$ and $H_{i,j}$.

For convenience, (5) is equivalently written in the following compact form:

$$\begin{cases} \xi(k+1) = F\xi(k) + Gy(k), \quad \xi(0) = 0, \\ \hat{x}(k) = H\xi(k), \end{cases} \tag{6}$$

with $\xi \in \mathbb{R}^{n_\xi}$, $y \in \mathbb{R}^{n_y}$ and $\hat{x} \in \mathbb{R}^{n_\xi}$, $n_\xi = nn_x$ and $n_y = n_1 + \cdots + n_n$, given by

$$\xi = [\xi_1' \dots \xi_n']', \qquad y = [y_1' \dots y_n']', \qquad \hat{x} = [\hat{x}_1' \dots \hat{x}_n']'$$

and F, G, and H are block-matrices with (i, j)-block, $i, j \in \mathscr{N}$, as follows:

$$\begin{cases} [F]_{ij} = F_{ij}, \quad [G]_{ij} = G_{ij}, \quad [H]_{ij} = H_{ij} : \text{ for } j \in \mathscr{N}_i, \\ [F]_{ij} = 0, \quad [G]_{ij} = 0, \quad [H]_{ij} = 0 : \text{ otherwise.} \end{cases}$$

Letting

$$\begin{cases} e_i := x - \hat{x}_i, \quad e = [e_1' \dots e_n']', \qquad \eta = [x' \ \xi']', \\ \widetilde{w} = [w' \ v']', \quad v = [v_1' \dots v_n']', \end{cases} \tag{7}$$

the dynamics of the overall estimation error, e, is given by

$$\begin{cases} \eta(k+1) = A_e\eta(k) + B_e\widetilde{w}(k), \\ e(k) = C_e\eta(k), \end{cases} \tag{8}$$

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