Automatica 72 (2016) 115–122

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive boundary observer for parabolic PDEs subject to domain and boundary parameter uncertainties ☆

Tarek Ahmed-Ali [a], Fouad Giri [a], Miroslav Krstic [b], Laurent Burlion [c], Françoise Lamnabhi-Lagarrigue [d]

[a] Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC, 14000 Caen, France

[b] Department of Mechanical and Aerospace Engineering, University of California at San Diego, La Jolla, CA 92093-0411, USA

[c] ONERA, The French Aerospace Lab, 31055 Toulouse, France

[d] LSS-CNRS, SUPELEC, EECI, 91192 Gif-sur-Yvette, France

ARTICLE INFO

Article history:
Received 20 July 2015
Received in revised form
31 January 2016
Accepted 19 May 2016

Keywords:
Parabolic PDEs
Parameter uncertainty
Adaptive observer
Backstepping design

ABSTRACT

We are considering the problem of state observation for a class of infinite dimensional systems modeled by parabolic type PDEs. The model is subject to parametric uncertainty entering in both the domain equation and the boundary condition. An adaptive boundary observer, providing online estimates of the system state and parameters, is designed using finite- and infinite-dimensional backstepping-like transformations. The observer is exponentially convergent under an ad hoc persistent excitation condition.

1. Introduction

The problem of observer design for infinite dimensional systems (IDSs) is given an increasing interest. Several observer design methods have been developed including the infinite dimensional Luenberger observer for linear IDSs (e.g. Amann, 1989; Curtain & Zwart, 1995; Lasiecka & Triggiani, 2000), the boundary observer design of bilinear IDSs (e.g. Bounit & Hammouri, 1997; Smyshlyaev & Krstic, 2005; Vries, Keesman, & Zwart, 2007; Xu, Ligaius, & Gauthier, 1995), backstepping-like boundary observers for parabolic partial integro-differential systems (Smyshlyaev & Krstic, 2005), initial state recovery in finite time of linear and semi-linear IDSs using forward and backward observers sequences (Fridman, 2013; Ramdani, Tucsnak, & Weiss, 2010; Tucsnak & Weiss, 2009), sampled-data (in time and space) observers of semilinear parabolic systems designed using Lyapunov functions and LMIs (Fridman & Blighovsky, 2012). Another important problem in system control is one of estimating unknown parameters. In the case of stable IDSs, this issue can be coped with in open-loop using parameter identification, using variants of the least-squares technique, see Smyshlyaev, Orlov, and Krstic (2009) and references there in. In the case of unstable systems, online parameter estimation is generally involved as part of adaptive controllers (Guo & Guo, 2013; Smyshlyaev & Krstic, 2007a,b). Most adaptive controllers rely on full state measurements, e.g. Bentsman and Orlov (2001), Bohm, Demetriou, Reich, and Rosen (1998), Guo and Guo (2013), Smyshlyaev and Krstic (2007a) and Smyshlyaev and Krstic (2008). Output-feedback adaptive controllers have been proposed for some classes of IDSs including specific parabolic PDEs (Hong & Bentsman, 1994; Smyshlyaev & Krstic, 2007b) and wave PDEs subject to a boundary harmonic disturbance linearly parameterized along a known set of functions (Smyshlyaev & Krstic, 2006). Also, in most adaptive controllers, the asymptotic convergence of the parameter estimates to their true values is not guaranteed. A quite complete description of the literature on adaptive controllers of IDSs systems described by parabolic equations, where both sensing and actuation are performed at the boundary and the unknown parameters are allowed to be spatially varying, can be found in Smyshlyaev and Krstic (2010).

This study is focused on the problem of designing adaptive observers featuring exponential convergence of the state estimate

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael Cantoni under the direction of Editor Richard Middleton.

E-mail addresses: tarek.ahmed-ali@ensicaen.fr (T. Ahmed-Ali), fouad.giri@unicaen.fr (F. Giri), krstic@ucsd.edu (M. Krstic), laurent.burlion@onera.fr (L. Burlion), francoise.lamnabhi-lagarrigue@lss.supelec.fr (F. Lamnabhi-Lagarrigue).

http://dx.doi.org/10.1016/j.automatica.2016.06.006

and the unknown parameter vector estimate. The problem has recently been addressed in Ahmed-Ali, Giri, Krstic, Burlion, and Lamnabhi-Lagarrigue (2015a); Ahmed-Ali, Giri, Krstic, Lamnabhi-Lagarrigue, and Burlion (2015b) for a class of semilinear parabolic PDEs. In Ahmed-Ali et al. (2015a), an exponentially convergent adaptive observer has been proposed for parabolic PDEs containing a single unknown parameter in the boundary condition. The result of Ahmed-Ali et al. (2015b) is an exponentially convergent adaptive observer of a class of semilinear parabolic PDE subject to domain parameter uncertainty. But, the number of unknown parameters must be equal to the number of available sensors in the domain. That is, if a single boundary sensor is available, only one unknown parameter is allowed to be in the domain. The novelty of the present study is twofold: (i) the parameter uncertainty is allowed to be both in the domain and at the boundary condition; (ii) the domain uncertainty is captured through an unknown parameter vector of arbitrary finite dimension, while only a single boundary sensing is available. It turns out that the adaptive observer problems addressed in Ahmed-Ali et al. (2015a,b) are particular cases of the problem considered here, whenever a single boundary sensing is available. Compared to earlier works on adaptive control or parameter identification (e.g. Guo & Guo, 2013; Smyshlyaev & Krstic, 2007a,b; Smyshlyaev et al., 2009), the present study involves a much wider class of systems, see Remark 1 hereafter. Furthermore, the new adaptive observer enjoys, under an ad hoc persistent excitation condition, exponential convergence while the earlier adaptive identifiers ensure L_2 convergence results. The parameter adaptive law is derived by using a finite-dimensional backstepping-like transformation, as in Ahmed-Ali et al. (2015a,b), and the observer domain varying gain is obtained by using an infinite-dimensional backstepping-like transformation, as in Smyshlyaev and Krstic (2005).

The paper is organized as follows: the observation problem statement is described in Section 2; the adaptive observer design and analysis are dealt with in Sections 3 and 4, respectively; a numerical simulation is made in Section 5; a conclusion and a reference list end the paper.

Notation. Throughout the paper, $\mathbf{R}^n$ is the n dimensional real space and $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices. The corresponding Euclidean norms are denoted $|\,|$. $L^2[0, 1]$ is the Hilbert space of square integrable functions defined on the interval $[0, 1]$ and $\|\cdot\|$ is the associated L^2-norm. $H^1(0, 1)$ is the Sobolev space of absolutely continuous functions $\eta : [0, 1] \to \mathbf{R}$ with $d\eta/d\varsigma \in L^2[0, 1]$. Given a function $w : [0, 1] \times \mathbf{R}_+ \to \mathbf{R}$; $(x, t) \to w(x, t)$, the notation $w[t]$ and $w_x[t]$ refer to the functions defined on $0 \le x \le 1$ by $(w[t])(x) = w(x, t)$ and $(w_x[t])(x) = \partial w(x, t)/\partial x$.

2. Observation problem statement

The system under study is described by a parabolic PDE of the following form:

$$u_t(x,t) = u_{xx}(x,t) + \phi(x,t)^T q_1, \quad 0 < x < 1,\ t > 0 \tag{1a}$$

with the following boundary condition:

$$u_x(0,t) = -q_0 u(0,t), \quad t \ge 0 \tag{1b}$$

where ϕ is a known function of class $C^1([0, 1] \times [0, \infty); \mathbf{R}^n)$, $q_1 \in \mathbf{R}^n$ and $q_0 \in \mathbf{R}$ are unknown vector and scalar parameters, respectively. For convenience, the following extended parameter vector is introduced:

$$\theta = \begin{bmatrix} q_0 \\ q_1 \end{bmatrix} \in \mathbf{R}^{n+1}. \tag{1c}$$

The goal is to generate accurate online estimates, $\hat{u}(x, t)$ and $\hat{\theta}(t)$, of the system distributed state $u(x, t)$ $(0 \le x \le 1; t \ge 0)$ and the parameter vector θ, based only on the input and output measurements $(U(t), y(t);\ \ t \ge 0)$ with

$$U(t) = u(1,t), \quad t \ge 0 \text{ (control signal)} \tag{1d}$$

$$y(t) = u(0,t), \quad t \ge 0 \text{ (system output)}. \tag{1e}$$

To achieve this objective, it is supposed that the state variable $u(x, t)$ $(0 \le x \le 1;\ t \ge 0)$ is bounded.

Remark 1. (1) In Smyshlyaev et al. (2009), two special forms belonging to the class defined by (1a)–(1b) have been considered. The first corresponds to the case where $q_0 = 0$, $q_1 \in \mathbf{R}$, and $\phi(x, t) = u(0, t)$. Then, the system involves a single uncertain parameter in the domain. The second special form is such that $q_0 \in \mathbf{R}$, $q_1 = 0$ and $\phi(x, t) = 0$, leading to a single uncertain parameter in the boundary. This second case has also been considered in Ahmed-Ali et al. (2015a) where an adaptive observer was developed.

(2) The class of systems (1a)–(1b) is also quite different from the one studied in Ahmed-Ali et al. (2015b). In the latter, it is supposed that a finite number of sensors are placed in the domain, while only one sensor is required here. It turns out that, in the case where the system in Ahmed-Ali et al. (2015b) contains a single sensor placed at the boundary then it falls in the class (1a)–(1b) with $q_0 = 0$, $q_1 \in \mathbf{R}$, and $\phi(x, t) = \psi(u(0, t), t)$.

3. Adaptive observer design

3.1. Observer structure

The system model (1a)–(1e) suggests the following observer structure:

$$\begin{aligned} \hat{u}_t(x,t) &= \hat{u}_{xx}(x,t) + \hat{q}_1^T(t)\phi(x,t) \\ &\quad - K(x)(\hat{u}(0,t) - y(t)) + v(x,t) \end{aligned} \tag{2a}$$

$$\hat{u}_x(0,t) = -\hat{q}_0(t)u(0,t) \tag{2b}$$

$$\hat{u}(1,t) = U(t) \tag{2c}$$

where $K(x)$ is a (space-dependent) observer gain, $\hat{q}_0 \in \mathbf{R}$, $\hat{q}_1 \in \mathbf{R}^n$ are parameter estimates, and $v(x, t)$ is an additional feedback term that will be determined latter. Let us introduce the following usual estimation errors:

$$\tilde{u}(x,t) = \hat{u}(x,t) - u(x,t) \quad \text{(state estimation error)} \tag{3a}$$

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta \overset{def}{=} \begin{bmatrix} \tilde{q}_0(t) \\ \tilde{q}_1(t) \end{bmatrix} \quad \text{(parameter estimation error)} \tag{3b}$$

where $\hat{\theta}(t) = \begin{bmatrix} \hat{q}_0(t) & \hat{q}_1(t) \end{bmatrix}^T$ and

$$\tilde{q}_0(t) = \hat{q}_0(t) - q_0, \qquad \tilde{q}_1(t) = \hat{q}_1(t) - q_1. \tag{3c}$$

Then, subtracting Eqs. (1a) to (2a), it follows that $\tilde{u}(x, t)$ satisfies the following equation:

$$\tilde{u}_t(x,t) = \tilde{u}_{xx}(x,t) + \tilde{q}_1^T(t)\phi(x,t) - K(x)\tilde{u}(0,t) + v(x,t) \tag{4a}$$

with the following boundary conditions, obtained using (1b), (1d) and (2b)–(2c):

$$\tilde{u}_x(0,t) = -\tilde{q}_0(t)u(0,t) \tag{4b}$$

$$\tilde{u}(1,t) = 0. \tag{4c}$$