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Brief paper

Linear interval observers under delayed measurements and delay-dependent positivity☆

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ABSTRACT

New interval observers are designed for linear systems with time-varying delays in the case of delayed measurements. Interval observers employ positivity and stability analysis of the estimation error system, which in the case of delayed measurements should be delay-dependent. New delay-dependent conditions of positivity for linear systems with time-varying delays are introduced. Efficiency of the obtained solution is demonstrated by examples.

1. Introduction

An estimation in nonlinear delayed systems is rather complicated (Fridman, 2014; Sipahi, Niculescu, Abdallah, Michiels, & Gu, 2011), as well as analysis of functional differential equations (Richard, 2003). Especially the observer synthesis is problematical for the cases when the model of a nonlinear delayed system contains parametric and/or signal uncertainties, or when the delay is time-varying and/or uncertain (Briat, Sename, & Lafay, 2011; Califano, Marquez-Martinez, & Moog, 2011; Zheng, Barbot, Boutat, Floquet, & Richard, 2011), the frequent applications include biosystems and chemical processes. Delayed measurements usually also increase complexity of estimators, which is a case in networked systems. An observer solution for these more complex situations is highly demanded in these and many others applications. Interval or set-membership estimation is a promising framework to observation in uncertain systems (Gouzé, Rapaport, & Hadj-Sadok, 2000; Jaulin, 2002; Kieffer & Walter, 2004; Mazenc & Bernard, 2011; Moisan, Bernard, & Gouzé, 2009; Raïssi, Efimov, & Zolghadri, 2012), when all uncertainty is included in the corresponding intervals or polytopes, and as a result the set of admissible values (an interval) for the state is provided at each instant of time.

In this work an interval observer for time-delay systems with delayed measurements is proposed. A peculiarity of an interval observer is that in addition to stability conditions, some restrictions on positivity of estimation error dynamics have to be imposed (in order to envelop the system solutions). The existing solutions in the field (Efimov, Perruquetti, & Richard, 2013; Efimov, Polyakov, & Richard, 2015; Mazenc, Niculescu, & Bernard, 2012; Polyakov, Efimov, Perruquetti, & Richard, 2013) are based on the delay-independent conditions of positivity from Ait Rami (2009) and Haddad and Chellaboina (2004). Some results on interval observer design for uncertain time-varying delay can be found in Efimov et al. (2013) and Ait Rami, Schönlein, and Jordan (2013). The first objective of this work is to use the delay-dependent positivity conditions (Efimov, Polyakov, Fridman, Perruquetti, & Richard, 2015), which are based on the theory of non-oscillatory solutions for functional differential equations (Agarwal, Berezansky, Braverman, & Domoshnitsky, 2012; Domoshnitsky, 2008). Next, two

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interval observers are designed for linear systems with delayed measurements (with time-varying delays) in the case of observable and detectable systems (with respect to (Efimov et al., 2015) the present work contains new result, Theorem 12, relaxed Assumption 1, and new examples). Efficiency of the obtained interval observers is demonstrated on a benchmark example from Mazenc et al. (2012) and a delayed nonlinear pendulum.

The paper is organized as follows. Some preliminaries and notation are given in Section 2. The delay-dependent positivity conditions are presented in Section 3. The interval observer design is performed for a class of linear time-delay systems (or a class of nonlinear systems in the output canonical form) with delayed measurements in Section 4. Examples of numerical simulation are presented in Section 5.

2. Preliminaries

2.1. Notation

- $\mathbb{R}$ is the Euclidean space ($\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$), $\mathcal{C}_\tau^n = C([-\tau, 0], \mathbb{R}^n)$ is the set of continuous maps from $[-\tau, 0]$ into $\mathbb{R}^n$ for $n \geq 1$; $\mathcal{C}_{\tau+}^n = \{y \in \mathcal{C}_\tau^n : y(s) \in \mathbb{R}_+^n,\ s \in [-\tau, 0]\}$;
- x_t is an element of $\mathcal{C}_\tau^n$ defined as $x_t(s) = x(t+s)$ for all $s \in [-\tau, 0]$;
- $|x|$ denotes the absolute value of $x \in \mathbb{R}$, $\|x\|_2$ is the Euclidean norm of a vector $x \in \mathbb{R}^n$, $\|\varphi\| = \sup_{t\in[-\tau,0]} \|\varphi(t)\|_2$ for $\varphi \in \mathcal{C}_\tau^n$;
- for a measurable and locally essentially bounded input $u : \mathbb{R}_+ \to \mathbb{R}^p$ the symbol $\|u\|_{[t_0,t_1)}$ denotes its L_∞ norm $\|u\|_{[t_0,t_1)} = ess\sup\{\|u(t)\|_2,\ t \in [t_0, t_1)\}$, the set of all such inputs $u \in \mathbb{R}^p$ with the property $\|u\|_{[0,+\infty)} < \infty$ will be denoted as $\mathcal{L}_\infty^p$;
- for a matrix $A \in \mathbb{R}^{n\times n}$ the vector of its eigenvalues is denoted as $\lambda(A)$;
- I_n and $0_{n\times m}$ denote the identity and zero matrices of dimensions $n \times n$ and $n \times m$, respectively;
- $a\,\mathcal{R}\,b$ corresponds to an elementwise relation $\mathcal{R} \in \{<, >, \leq, \geq\}$ (a and b are vectors or matrices): for example $a < b$ (vectors) means $\forall i : a_i < b_i$; for $\phi, \varphi \in \mathcal{C}_\tau$ the relation $\phi\,\mathcal{R}\,\varphi$ has to be understood elementwise for whole domain of definition of the functions, *i.e.* $\phi(s)\,\mathcal{R}\,\varphi(s)$ for all $s \in [-\tau, 0]$;
- for a symmetric matrix Υ, the relation $\Upsilon \prec 0$ ($\Upsilon \preceq 0$) means that the matrix is negative (semi) definite.

2.2. Delay-independent conditions of positivity

Consider a time-invariant linear system with time-varying delay:

$$\dot{x}(t) = A_0 x(t) - A_1 x(t - \tau(t)) + b(t), \quad t \geq 0, \tag{1}$$
$$x(\theta) = \phi(\theta) \quad \text{for } -\overline{\tau} \leq \theta \leq 0,\ \phi \in \mathcal{C}_\tau^n,$$

where $x(t) \in \mathbb{R}^n$, $x_t \in \mathcal{C}_{\overline{\tau}}^n$ is the state function; $\tau : \mathbb{R}_+ \to [-\overline{\tau}, 0]$ is the time-varying delay, a Lebesgue measurable function of time, $\overline{\tau} \in \mathbb{R}_+$ is the maximum delay; $b \in \mathcal{L}_\infty^n$ is the input; the constant matrices A_0 and A_1 have appropriate dimensions. The matrix A_0 is called Metzler if all its off-diagonal elements are nonnegative. The system (1) is called *positive* if for $x_0 \geq 0$ it has the corresponding solution $x(t) \geq 0$ for all $t \geq 0$.

Lemma 1 (*Ait Rami, 2009* and *Haddad & Chellaboina, 2004*)**.** *The system (1) is positive iff A_0 is Metzler, $A_1 \leq 0$ and $b(t) \geq 0$ for all $t \geq 0$. A positive system (1) is asymptotically stable for $b(t) \equiv 0$ for all $\overline{\tau} \in \mathbb{R}_+$ iff there are $p, q \in \mathbb{R}_+^n$ ($p > 0$ and $q > 0$) such that*

$$p^{\mathrm{T}}[A_0 - A_1] + q^{\mathrm{T}} = 0.$$

Under conditions of the above lemma the system has bounded solutions for $b \in \mathcal{L}_\infty^n$. Note that for linear time-invariant systems the conditions of positive invariance of polyhedral sets have been similarly given in Dambrine, Richard, and Borne (1995), as well as conditions of asymptotic stability in the nonlinear case have been considered in Borne, Dambrine, Perruquetti, and Richard (2003) and Dambrine and Richard (1993, 1994).

3. Delay-dependent conditions of positivity

Consider a scalar time-varying linear system with time-varying delays (Agarwal et al., 2012):

$$\dot{x}(t) = a_0(t)x[g(t)] - a_1(t)x[h(t)] + b(t), \tag{2}$$
$$x(\theta) = 0 \quad \text{for } \theta < 0,\ x(0) \in \mathbb{R}, \tag{3}$$

where $a_0 \in \mathcal{L}_\infty, a_1 \in \mathcal{L}_\infty, b \in \mathcal{L}_\infty, h(t) - t \in \mathcal{L}_\infty, g(t) - t \in \mathcal{L}_\infty$ and $h(t) \leq t, g(t) \leq t$ for all $t \geq 0$. For the system (2) the initial condition in (3) is, in general, not a continuous function (if $x(0) \neq 0$).

The following result proposes delay-independent positivity conditions.

Lemma 2 (*Agarwal et al., 2012, Corollary 15.7*)**.** *Let $h(t) \leq g(t)$ and $0 \leq a_1(t) \leq a_0(t)$ for all $t \geq 0$. If $x(0) \geq 0$ and $b(t) \geq 0$ for all $t \geq 0$, then the corresponding solution of (2), (3) $x(t) \geq 0$ for all $t \geq 0$.*

Recall that in this case positivity is guaranteed for "discontinuous" initial conditions. The peculiarity of the condition $0 \leq a_1(t) \leq a_0(t)$ is that it may correspond to an unstable system (2). In order to overcome this issue, delay-dependent conditions can be introduced.

Lemma 3 (*Agarwal et al., 2012, Corollary 15.9*)**.** *Let $h(t) \leq g(t)$ and $0 \leq \frac{1}{e}a_0(t) \leq a_1(t)$ for all $t \geq 0$ with*

$$\sup_{t\in\mathbb{R}_+} \int_{h(t)}^{t} \left[a_1(\xi) - \frac{1}{e}a_0(\xi)\right] d\xi < \frac{1}{e},$$

where $e = \exp(1)$. If $x(0) \geq 0$ and $b(t) \geq 0$ for all $t \geq 0$, then $x(t) \geq 0$ for all $t \geq 0$ in (2), (3).

These lemmas describe positivity conditions for the system (2), (3), which is more complex than (1), but scalar, they can also be extended to the n-dimensional system (1).

Corollary 4. *The system (1) with $b(t) \geq 0$ for all $t \geq 0$ and initial conditions*

$$x(\theta) = 0 \quad \text{for } -\overline{\tau} \leq \theta < 0,\ x(0) \in \mathbb{R}_+^n,$$

is positive if $-A_1$ is Metzler, $A_0 \geq 0$, and

$$0 \leq (A_0)_{i,i} \leq e(A_1)_{i,i} < (A_0)_{i,i} + \overline{\tau}^{-1}$$

for all $i = 1, \ldots, n$.

From these corollaries it is easy to conclude that the delay-dependent case studied in Lemmas 2 and 3 is crucially different from the delay-independent positivity conditions given first in Lemma 1, where in the scalar case the restriction $a_1 \leq 0$ implies positivity of (1) and the condition $a_0 < a_1$ according to Lemma 1 ensures stability for any $\overline{\tau}$. These results do not contradict to Remark 3.1 of Haddad and Chellaboina (2004), since $x(\theta) \neq 0$ for $-\tau \leq \theta < 0$ there. A graphical illustration of different delay-independent conditions (positivity from Lemmas 1 and 2) and delay-dependent ones (from Lemma 3, the stability conditions are also satisfied in this case) for the system (2) is given in Fig. 1 in the plane (a_0, a_1). It is worth stressing that an extension of the positivity domain in Lemma 3 is also achieved due to restrictions imposed on initial conditions in (3).