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Brief paper

A hybrid scheme for reducing peaking in high-gain observers for a class of nonlinear systems[☆]

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ABSTRACT

For a family of nonlinearly interconnected second order plants, we propose a hybrid modification of the well known high-gain observer design, which leads to guaranteed asymptotic stability properties of the error dynamics, as well as exhibiting significantly reduced peaking. The hybrid modification results in suitable jumps of the observer state, which therefore corresponds to a discontinuous function of ordinary time. The jump conditions depend on the plant output and on extra states incorporated in the observer that evolve according to some suitable combination of the output error and of its integral. Our result is illustrated on a well known case study taken from the literature, and also on a planar robot with rigid links.

1. Introduction

The observer design problem for nonlinear dynamical systems has attracted the attention of several researchers over the last years (see the books Besançon, 2007; Gauthier & Kupka, 2001 and references therein). Different approaches are possible such as high-gain observers (Esfandiari & Khalil, 1992; Gauthier, Hammouri, & Othman, 1992; Khalil & Praly, 2013) nonlinear Luenberger observers (see Andrieu & Praly, 2006), passivity (as in Ailon & Ortega, 1993) or bounds on the slope of the nonlinearity (see e.g. Arcak & Kokotovic, 2001). High-gain observers design dates back to the late 1980s (Esfandiari & Khalil, 1992; Gauthier et al., 1992; Nicosia, Tomei, & Tornambé, 1989) and essentially corresponds to the intuitive idea that a very strong Lyapunov decrease arising from high-gain linear output injection is capable of dominating nonlinear terms satisfying a suitable bound. Due to this intrinsic nature of the high-gain approach, these observers suffer from the so-called peaking phenomenon (Esfandiari & Khalil, 1992; Khalil, 1999; Kokotovic, 1992; Sepulchre, Janković, & Kokotović, 1997) wherein the observer error can grow very large during the transient response. One of the interests of asymptotic observers is to use the state estimation in an output feedback loop to achieve asymptotic stabilization. To protect the system from the destabilizing effect of peaking, high-gain observers have to be followed by saturation as explained in Esfandiari and Khalil (1992). This strategy has been formalized in Teel and Praly (1994) in combination with an a priori knowledge of a set of initial conditions to achieve semiglobal stabilization and in Kaliora, Astolfi, and Praly (2006) in combination with a norm observer. Oliveira, Peixoto, Costa, and Hsu (2010); Oliveira, Peixoto, and Hsu (2013) successively exploit a dwell-time control activation to avoid the peaking phenomenon.

In this paper, we focus on the state estimation problem and we do not assume that an estimation of the norm of the state is available. We revisit high-gain observer designs within the hybrid dynamical systems framework of Goebel, Sanfelice, and Teel (2009, 2012) and, for a family of nonlinearly interconnected second order plants, we propose a hybrid high-gain observer having the novel feature of not exhibiting peaking when the (high) gain of the observer is increased. In particular, the observers that we propose comprise a flow dynamics (continuous-time evolution) which essentially coincides with the original continuous-time high-gain

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observer of Esfandiari and Khalil (1992), Khalil (1999) and Nicosia et al. (1989), augmented with a suitable resetting rule enforced on the observer state. Such a scheme ensures that trajectories approaching a region of the state space where peaking occurs, are projected in another region where peaking is absent. We show that the Lyapunov function establishing exponential stability of the error system does not increase, thus preserving the asymptotic state estimation property of the observer. Alternative hybrid observation schemes have been presented in, e.g., Paesa, Franco, Llorente, Lopez-Nicolas, and Saguez (2012), Raff and Allgöwer (2008) and references therein, which do not specifically focus on peaking reduction.

Preliminary results in the directions of this paper have been presented in Prieur, Tarbouriech, and Zaccarian (2012). Here, as compared to those preliminary results, we extend our theory from a single second-order plant to a family of nonlinearly interconnected second-order plant, which allows us to address the case of joint speed estimation in n-degrees of freedom Euler–Lagrange systems. Moreover, we propose different resetting laws and jump and flow sets, to allow for a suitable adaptation of the results to the $2n$-dimensional case. Another observer approach to avoid overshoot in the case of planar multidimensional systems is the reduced order observer. More precisely, this framework presented in Besançon (2000, Lemma 3.1) computes a reduced order observer for any system for which a high-gain observer has been computed and thus leads to a reduction of the overshoot. In the context of planar observer (that is $n = 1$ in (1)), the obtained observer is scalar and has a decreasing state estimation error. However the estimate has a feedthrough term from the measurement output and thus is more sensitive to measurement noise. The hybrid observer that is designed in the present paper can be seen as a tradeoff between these two techniques. Recently in Astolfi and Praly (2013) another approach to avoid peaking has been investigated. It employs a projection approach and requires the knowledge of a convex set in which the state trajectory remains. This is not required in our approach.

The paper is organized as follows. Section 2 is dedicated to revise the high-gain observer by introducing suitable resetting laws for the state of the observer in order to reduce the peaking. Section 3 deals with an application of the results to a robotic example.

Notation. Throughout the paper, notation is standard. $|\cdot|$ stands for the Euclidean norm. For any vector x (resp. matrix A), x^T (resp. matrix A^T) denotes its transpose.

2. A hybrid observer with reduced peaking

Before considering a hybrid observer in Section 2.2, let us first define the class of nonlinear planar systems and the associated high-gain observer. The peaking phenomenon is also briefly discussed and illustrated.

2.1. High-gain observer design and Lyapunov properties

Following the standard approach in, e.g., Esfandiari and Khalil (1992), we consider a set of n second order nonlinear systems whose state is denoted by $x = (x_1, \ldots, x_n) \in \mathbb{R}^{2n}$ and $x_i = (p_i, v_i) \in \mathbb{R}^2$, where each substate x_i obeys the following second order nonlinear dynamics with nonlinear couplings arising from the terms ψ_i and ϕ_i:

$$\begin{aligned} \dot{p}_i &= v_i + \psi_i(y), \qquad \dot{v}_i = \phi_i(x, u) \\ y_i &= p_i, \end{aligned} \tag{1}$$

for all $i = 1, \ldots, n$, where $y = (y_1, \ldots, y_n) \in \mathbb{R}^n$ is the measurement output, $u \in \mathbb{R}^p$ is a known control input that is piecewise continuous[1] and ψ_i and ϕ_i, $i = 1, \ldots, n$ are known nonlinearities establishing a nonlinear coupling among the n planar systems. For example, in the robot example of Section 3, these couplings are caused by strong nonlinear coupling effects between the two joints.

Remark 1. Note that in Esfandiari and Khalil (1992) an output feedback is designed assuming that the nonlinearities ϕ_i are uncertain. In this paper, since we address the asymptotic observer design problem we consider the case in which dynamics are perfectly known (as Besançon, 2007 or Gauthier & Kupka, 2001 for instance). ◇

High-gain observers are used to provide an estimate $\hat{x}$ of the state x. If the following Lipschitz-like condition holds for a suitable constant $L_\delta > 0$:

$$|\phi_i(x, u) - \phi_i(\hat{x}, u)| \le L_\delta |x - \hat{x}|, \quad \forall i = 1, \ldots, n, \tag{2}$$

for all x and $\hat{x}$ in $\mathbb{R}^{2n}$, u in $\mathbb{R}^p$, then a high-gain observer can be designed to ensure asymptotic stability of the origin of the feedback system from the observed state.

Remark 2. Lipschitz condition (2) is standard in the context of high-gain observer designs. Note however that it has been relaxed in Lei, Wei, and Lin (2005) allowing for a strictly increasing high-gain parameter. We do not follow this route since this approach may lead to undesirable behavior in the context of disturbed measurements. Moreover, this Lipschitz condition has been also slightly relaxed in Andrieu, Praly, and Astolfi (2009) employing homogeneous tools. Possible extensions of our tools in these directions are object of future work. ◇

For system (1) we can design the following high-gain observer:

$$\dot{\hat{x}}_i = \begin{bmatrix} \dot{\hat{p}}_i \\ \dot{\hat{v}}_i \end{bmatrix} = \begin{bmatrix} \hat{v}_i + \psi_i(y) + \ell k_1 e_{pi} \\ \phi_i(\hat{x}, u) + \ell^2 k_2 e_{pi} \end{bmatrix} =: f_i(\hat{x}, e_{pi}, u), \tag{3}$$

$i = 1, \ldots, n$, where $e_{pi} = y_i - \hat{p}_i$ is the ith output error, k_1 and k_2 are any pair of positive scalars, and ℓ is the "high" gain of the high-gain observer. The following dynamics can be easily derived from (1), (3) if one defines the scaled error coordinates $e_i := (e_{pi}, e_{vi}) := (p_i - \hat{p}_i, \ell^{-1}(v_i - \hat{v}_i))$:

$$\begin{aligned} \dot{e}_i &= \ell \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} e_i + \begin{bmatrix} 0 \\ \dfrac{\phi_i(x, u) - \phi_i(\hat{x}, u)}{\ell} \end{bmatrix} \\ &=: \ell A_e e_i + \begin{bmatrix} 0 \\ \delta_i(x, \hat{x}, u) \end{bmatrix}, \quad i = 1, \ldots, n, \end{aligned} \tag{4}$$

which reveals the potential for the high-gain $\ell \gg 1$ to dominate over each nonlinear term δ_i, $i = 1, \ldots, n$. In particular, from (2), if $\ell \ge 1$, we obtain

$$|\delta_i(x, \hat{x}, u)| \le \frac{L_\delta}{\ell} |x - \hat{x}| \le L_\delta |e|. \tag{5}$$

A possible way to characterize the set of sufficiently high-gains ℓ ensuring exponential stability of the error dynamics (4) is given in the next proposition, which is given here for completeness and will be useful for our results.

[1] The assumption on piecewise continuity of input u could be relaxed but special care has to be paid to the hybridization of the corresponding function of time (see, e.g., Sanfelice, 2014 for details).