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Brief paper

Synchronization of linear dynamical networks on time scales: Pinning control via delayed impulses[☆]

Xinzhi Liu[1], Kexue Zhang

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

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ABSTRACT

This paper studies the synchronization problem of complex dynamical networks (CDNs) on time scales. A pinning impulsive control scheme that takes into account of time-delay effects is designed to achieve synchronization of CDNs on time scales with the state of an isolated node. Based on the theory of time scales and the direct Lyapunov method, a synchronization criterion is established for linear CDNs on general time scales. Our result shows that, by impulsive control a small portion of nodes, the consensus of CDNs on time scales can be achieved. According to our pinning impulsive control scheme, different numbers of nodes will be selected at each impulsive instant and time delay is considered in the pinning impulses. The modeling framework developed in this paper is a unification and generalization of many existing continuous-time and discrete-time CDN models, while the pinning impulsive control scheme is an extension of the existing control scheme for synchronization of continuous-time CDNs. Moreover, the idea of studying dynamical systems on time scales provide a unified approach to investigate continuous-time system and its discrete-time counterpart simultaneously. Numerical simulations are given to illustrate the effectiveness of the theoretical analysis.

1. Introduction

Complex dynamical networks (CDNs), which consist of a large set of interconnected nodes with each node being a fundamental unit with detailed contents, have received a great deal of attention from various disciplines (see, e.g., Barabasi & Albert, 2002; Strogatz, 2001; Wang & Chen, 2001), such as biological, mathematical, physical, social and engineering sciences. As one of the most interesting and significant phenomena in complex dynamical networks, synchronization of a group of dynamical nodes in a complex network topology has been investigated intensively (see Pecora, Carroll, Johnson, Mar, & Fink, 2000; Wang & Chen, 2002a), and a wide variety of control methods have been successfully utilized to realize the synchronization of CDNs (see, e.g., Fradkov & Junussov, 2013; Liu, Liu, & Chen, 2005; Long, Wu, & Liu, 2005; Lu, Kurths, & Cao, 2012; Mahdavi, Menhaj, Kurths, Lu, & Afshar, 2012; Zhang, Lu, & Zhao, 2010).

During the past decades, the method of impulsive control has been successfully used for synchronization of both continuous and discrete complex dynamical networks (see, e.g., Liu et al., 2005; Long et al., 2005; Zhang et al., 2010). It is clear to see that the continuous and discrete networks are normally investigated separately, and the results concerning discrete CDNs are carried quite easily from the corresponding results from their continuous counterparts. Therefore, it is natural to consider whether it is possible to provide a framework to study both the continuous and discrete CDNs simultaneously. On the other hand, from the modeling and numerical points of view (see, e.g., Atici & Uysal, 2008; Seiffertt, Sanyal, & Wunsch, 2008), it is more realistic to model a network by dynamical network which incorporates both continuous and discrete times. The recently developed theory of time scales, which was initialed by Stefan Hilger in his Ph.D. thesis in 1988, offers the desired unified method. The purpose of this theory is to unify the existing theory of continuous and discrete dynamic systems, and extend these theories to dynamic systems on generalized hybrid (continuous/discrete) domains. The theory of time scales has gained much attention and is undergoing rapid development in diverse areas (see, e.g., Bohner, Fan, & Zhang, 2007; Marks, Gravagne, Davis, & DaCunha, 2006; Seiffertt et al., 2008).

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E-mail addresses: xzliu@uwaterloo.ca (X. Liu), k57zhang@uwaterloo.ca (K. Zhang).

[1] Fax: +1 519 746 4319.

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Recently, neural networks on time scales have attracted increasing interest, and stability and synchronization of different kinds of dynamical networks on time scales have been studied (see, e.g., Hu & Wang, 2013; Luk'yanova & Martynyuk, 2010). The traditional method to synchronize a dynamical network is to add a controller to each of the network nodes to tame the node dynamics to approach a desired synchronization trajectory. However, a dynamical network is normally composed of a large set of high dimensional nodes, and then it is expensive and infeasible to control all of the network nodes. Hinted by this practical consideration, the idea of controlling small portions of nodes named pinning control was introduced in Li, Wang, and Chen (2004) and Wang and Chen (2002b). Obviously, the pinning control method reduces the control cost to a certain extent by reducing the amount of controllers added to the nodes. It is worth noting that the control cost can be further reduced by combining the idea of pinning control and impulsive control concept, i.e., adding the impulsive controllers to small fractions of network nodes. Hence, the notion of pinning impulsive control came into people's vision and has stimulated many interesting pinning impulsive control strategies for synchronization of dynamical networks (see, e.g., Hu & Xu, 2009; Lu, Ho, Cao, & Kurths, 2013; Lu et al., 2012; Tang, Wong, & Fang, 2011; Zhou, Wu, & Xiang, 2011). However, as far as we know, the pinning impulsive synchronization problem of CDNs on time scales is still open.

Motivated by the aforementioned discussion, we shall investigate the synchronization of CDNs on time scales. Recently, systems with delayed impulses have been studied by many researchers mainly due to the wide existence of time delay (see, e.g., Cheng, Deng, & Yao, 2014; Chen & Zheng, 2011). It has been shown that the delayed impulses could help to stabilize the systems, and, on the other hand, could lead to poor dynamic performance of the systems. Therefore, it is essential to investigate systems subject to delayed impulses. In this paper, a pinning impulsive control scheme with time-delay effects is designed to achieve the synchronization of CDNs on general time scales. Based on the theory of time scales and the Lyapunov method, a synchronization criterion has been established for linear CDNs on time scales. To the best of our knowledge, the pinning delayed-impulsive synchronization problem of continuous/discrete CDNs has not been investigated yet. Hence, our result is not only applicable to continuous/discrete CDNs but also applicable to CDNs on hybrid time domains.

The outline of this paper is as follows. In Section 2, we introduce some basic knowledge for the theory of time scales. In Section 3, we formulate the problem of synchronization for linear CDNs on time scales, and propose the pinning delayed-impulsive control strategy. In Section 4, an impulsive synchronization criterion is established for linear CDNs on general time scales. In Section 5, numerical simulations are given to illustrate the effectiveness of the proposed control algorithm. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, we recall some basic definitions and properties of time scales which are used in what follows. Let $\mathbb{T}$ be a time scale (an arbitrary nonempty closed subset of the real number set $\mathbb{R}$). We assume that $\mathbb{T}$ is a topological space with relative topology induced from $\mathbb{R}$. If $a, b \in \mathbb{T}$, we then define the interval $[a, b]$ in $\mathbb{T}$ by $[a, b] := \{t \in \mathbb{T} : a \le t \le b\}$. Open intervals and half-open intervals etc. are defined accordingly. The mappings $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ defined as $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$ and $\rho(t) = \sup\{s \in \mathbb{T} : s < t\}$ are called forward and backward jump operators, respectively. A nonmaximal element $t \in \mathbb{T}$ is right-scattered if $\sigma(t) > t$ and right-dense if $\sigma(t) = t$. A nonminimal element $t \in \mathbb{T}$ is left-scattered if $\rho(t) < t$ and left-dense if $\rho(t) = t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T}\setminus\{m\}$, otherwise, $\mathbb{T}^k = \mathbb{T}$. The graininess function $\mu : \mathbb{T} \to \mathbb{R}^+$ is defined by $\mu(t) = \sigma(t) - t$.

Definition 2.1. For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $y(t)$, $y^{\Delta}(t)$, to be the number (when it exists) with the property that for any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t-\delta, t+\delta) \bigcap \mathbb{T}$ for some $\delta > 0$) such that $|y(\sigma(t)) - y(s) - y^{\Delta}(t)(\sigma(t) - s)| \le \varepsilon|\sigma(t) - s|$ for all $s \in U$.

A function $f : \mathbb{T} \to \mathbb{R}$ is rd-continuous provided it is continuous at right-dense points in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd} = C_{rd}(\mathbb{T}, \mathbb{R})$. If f is continuous at each right-dense point and each left-dense point, f is said to be continuous function on $\mathbb{T}$.

Definition 2.2. Let $f \in C_{rd}$. A function $g : \mathbb{T} \to \mathbb{R}$ is called the antiderivative of f on $\mathbb{T}$ if it is differentiable on $\mathbb{T}$ and satisfies $g^{\Delta}(t) = f(t)$ for $t \in \mathbb{T}$. In this case, we define $\int_a^t f(s)\Delta s = g(t) - g(a)$, where $t, a \in \mathbb{T}$.

We say that a function $p : \mathbb{T} \to \mathbb{R}$ is regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}$ hold. The set of all regressive and rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted in this paper by $\mathcal{R} = \mathcal{R}(\mathbb{T}, \mathbb{R})$, and the set of all positively regressive elements of $\mathcal{R}$ is denoted by $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0$ for all $t \in \mathbb{T}\}$.

Definition 2.3. If $p \in \mathcal{R}$, then we define the exponential function on time scale $\mathbb{T}$ by $e_p(t, s) = \exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\right)$, for $t, s \in \mathbb{T}$, where the cylinder transformation $\xi_h(z) = \begin{cases} \frac{\mathrm{Log}(1+hz)}{h}, & h \neq 0 \\ z, & h = 0 \end{cases}$, where Log is the natural logarithm function.

Remark 2.1. Let $\alpha \in \mathcal{R}$ be constant. If $\mathbb{T} = \mathbb{Z}$, then $e_\alpha(t, t_0) = (1+\alpha)^{t-t_0}$ for all $t \in \mathbb{T}$. If $\mathbb{T} = \mathbb{R}$, then $e_\alpha(t, t_0) = e^{\alpha(t-t_0)}$ for all $t \in \mathbb{T}$. If $\alpha \ge 0$, then $e_\alpha(t, s) \ge 1$ for $t \ge s$ and $t, s \in \mathbb{T}$. Moreover, for $t, s, r \in \mathbb{T}$, $e_\alpha(t, s) = \frac{1}{e_\alpha(s,t)}$ and $e_\alpha(t, r)e_\alpha(r, s) = e_\alpha(t, s)$, which will be used in the proof of main result in this paper.

In the sequel, we present two lemmas from Bohner and Peterson (2001) which will be essential to prove our main result.

Lemma 2.1. *If $f \in C_{rd}$ and $t \in \mathbb{T}^k$, then $\int_t^{\sigma(t)} f(\tau)\Delta\tau = \mu(t)f(t)$.*

Remark 2.2. If $p \in \mathcal{R}$ and $t \in \mathbb{T}^k$, then, from Definition 2.3 and Lemma 2.1, we have $e_p(\sigma(t), t) = 1 + \mu(t)p(t)$.

Lemma 2.2. *Let $f \in C_{rd}$ and $p \in \mathcal{R}^+$. Then, for all $t \in \mathbb{T}$, inequality $y^{\Delta}(t) \le p(t)y(t) + f(t)$ implies that $y(t) \le y(t_0)e_p(t, t_0) + \int_{t_0}^t e_p(t, \sigma(\tau))f(\tau)\Delta\tau$.*

3. Problem formulation

Throughout this paper, we make the following notations. Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space with the Euclidean norm $\|\cdot\|$. Let $\mathbb{R}^+ = [0, \infty)$, $\mathbb{Z}^+ = \{0, 1, 2, \ldots\}$ and $\mathbb{N} = \{1, 2, \ldots\}$. $\mathbb{R}^{n\times n}$ is the set of all $n \times n$ real matrices. The superscript 'T' stands for the transpose of a matrix. I is an appropriately dimensioned identity matrix. $\lambda_{\max}(P)$ denotes the largest eigenvalue of the symmetric matrix P. The notion $X \ge Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). Let $\mathbb{T}$ be a time scale with $t_0 = 0$ as minimal element and no maximal element.