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Componentwise ultimate bounds for positive discrete time-delay systems perturbed by interval disturbances☆

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ABSTRACT

This paper presents a method to derive componentwise ultimate upper bounds and componentwise ultimate lower bounds for linear positive systems with time-varying delays and bounded disturbances. The disturbance vector is assumed to vary within a known interval whose lower bound may be different from zero. We first derive a sufficient condition for the existence of componentwise ultimate bounds. This condition is given in terms of the spectral radius of the system matrices which is easy to check and allows us to compute directly both the smallest componentwise ultimate upper bound and the largest componentwise ultimate lower bound. Then, by using the comparison method, we extend the obtained result to a class of nonlinear time-delay systems which has linear positive bounds. Two numerical examples are given to illustrate the effectiveness of the obtained results.

1. Introduction

In general, it is hard (sometime impossible) to achieve asymptotic stability for dynamical systems perturbed by unknown-but-bounded disturbances. Instead, the convergence of the system's trajectories within a bounded set after a large enough time can be guaranteed. Such a set is called an ultimate bound set of the system (Khalil, 2002). The problem of finding the smallest possible ultimate bound set for perturbed systems has been an important topic in control engineering and has attracted considerable research attention (see, Corless & Leitmann, 1993; Haimovich, Kofman, & Seron, 2007; Haimovich & Seron, 2010; Khalil, 2002; Kofman, Haimovich, & Seron, 2007 and the references therein).

Recently, there is a growing interest in the problem of finding ultimate bound sets for perturbed systems with time delays. For linear time-delay systems whose matrices are constant, a widely used approach is based on the Lyapunov method combining with linear matrix inequality techniques. By using this approach, Fridman and Dambrine (2009), Han, Fridman, and Spurgeon (2010, 2012), Nam, Pathirana, and Trinh (2013, 2014, 2015a) and Oucheriah (2006) derived sufficient conditions for the existence of ellipsoidal ultimate bound sets. Another approach which is based on the comparison method combining with Metzler matrix or Schur matrix is also widely used (Haimovich et al., 2007; Haimovich & Seron, 2010, 2013, 2014; Kofman et al., 2007). To achieve smaller ultimate bound sets, Haimovich et al. (2007), Haimovich and Seron (2010, 2013, 2014) and Kofman et al. (2007) derived ultimate bound for each partial state vector, i.e. componentwise ultimate bounds. Hence, their ultimate bound sets are smaller than the ones derived by employing a norm for bounding the full state vector. Very recently, by estimating directly the state vector, Hien and Trinh (2014) and Xu and Ge (2015) derived componentwise ultimate bounds for general nonlinear time-delay systems. Note that, in all of the above papers, disturbances are considered under the assumption that their absolute value varies from zero to an upper bound. In practice, however, the lower bound of the absolute value of the disturbance vector may be not necessary to be zero.

Motivated by the above discussion, in this paper, we study the problem of finding componentwise ultimate bounds for linear positive systems with time-varying delays and bounded disturbances. Different from the existing results, the disturbance vector is assumed to vary within a known vector-valued interval whose lower bound may be different from zero. By estimating the

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system state, we first derive a sufficient condition for the existence of both componentwise ultimate upper bound and lower bound. The upper bound is shown to be smallest and the lower bound is shown to be largest. Then, based on the comparison method, we extend the obtained result to nonlinear time-delay systems which has linear positive bounds. Lastly, two numerical examples are given to illustrate the obtained results.

2. Notations and problem statement

Notations: $\mathbb{N}$ is the set of nonnegative integers; $\mathbb{R}^n(\mathbb{R}^n_{0,+}, \mathbb{R}^n_+)$ is n-dimensional (nonnegative, positive) vector space; For two vectors $x = [x_1\ x_2\ \cdots\ x_n]^T \in \mathbb{R}^n$, $y = [y_1\ y_2\ \cdots\ y_n]^T \in \mathbb{R}^n$, two $n \times n$-matrices $A = [a_{ij}]$, $B = [b_{ij}]$, notation $x \prec y(\preceq y)$ means that $x_i < y_i(\leq y_i), \forall i = 1, \ldots, n$; $A \prec B(\preceq B)$ means that $a_{ij} < b_{ij}(\leq b_{ij}), \forall i, j = 1, \ldots, n$; A is nonnegative if $0 \preceq A$; $\rho(A) = \max\{|\lambda| : \lambda \in \sigma(A)\}$ with $\sigma(A)$ being the spectrum of A and $|x| := [|x_1|\ |x_2|\ \cdots\ |x_n|]^T$; For two vectors $q_1 \preceq q_2$, $[q_1, q_2] := \{x \in \mathbb{R}^n : q_1 \preceq x \preceq q_2\}$ is a vector-valued interval; If $x(t)$ is a vector-valued function then $\limsup_{t\to\infty} x(t)$ (or $\liminf_{t\to\infty} x(t)$) denotes the vector obtained by taking $\limsup_{t\to\infty}$, ($\liminf_{t\to\infty}$) of component of $x(t)$.

Consider the following linear positive system with a time-varying delay and bounded disturbances

$$x(t+1) = A_0x(t) + A_1x(t-\tau_1(t)) + B\omega(t), \quad t \in \mathbb{N}, \tag{1}$$
$$x(s) = \upsilon(s), \quad s \in \{-h, -h+1, \ldots, 0\}$$

where $x(t) \in \mathbb{R}^n_{0,+}$ is the state vector; $\upsilon(s) \in \mathbb{R}^n_{0,+}$, $s \in \{-h, -h+1, \ldots, 0\}$ are initial values; $\omega(t) \in \mathbb{R}^k_{0,+}$ is the disturbance vector varying within a known interval, i.e.,

$$0 \preceq \underline{\omega} \preceq \omega(t) \preceq \overline{\omega}, \tag{2}$$

$\underline{\omega}, \overline{\omega}$ are known vectors; time-varying delay, $\tau_1(t) \in [0, h]$, is a given integer-valued function, h is a known integer; A_0, A_1 and B are nonnegative matrices.

Let us denote a solution with initial values $x(s) = \upsilon(s)$, $s \in \{-h, \ldots, 0\}$ and a disturbance vector $\omega(t)$ of system (1) by $x(t, \upsilon, \omega)$. Then, we have definitions of componentwise ultimate upper bound and lower bound of system (1) as follows:

Definition 1. (i) A nonnegative vector $\overline{q}$ is called a componentwise ultimate upper bound of system (1) if for any initial condition $\upsilon(s)$, $s \in \{-h, \ldots, 0\}$ and for any disturbance vector $\omega(t)$ satisfying (2), we have

$$\limsup_{t\to\infty} x(t, \upsilon, \omega) \preceq \overline{q};$$

(ii) Similarly, a nonnegative vector $\underline{q}$ is called a componentwise ultimate lower bound of system (1) if

$$\liminf_{t\to\infty} x(t, \upsilon, \omega) \succeq \underline{q}.$$

The main objective of this paper is to derive the smallest componentwise ultimate upper bound $\overline{q}$ and the largest componentwise ultimate lower bound $\underline{q}$, for system (1).

3. Main result

3.1. Componentwise ultimate bounds for linear positive systems

Let us consider the following two respective linear systems:

$$z(t+1) = A_0z(t) + A_1z(t-\tau_1(t)) + B\overline{\omega}, \quad t \in \mathbb{N} \tag{3}$$
$$z(s) = \psi(s), \quad s \in \{-h, -h+1, \ldots, 0\},$$
$$g(t+1) = A_0g(t) + A_1g(t-\tau_1(t)) + B\underline{\omega}, \quad t \in \mathbb{N} \tag{4}$$
$$g(s) = \phi(s), \quad s \in \{-h, -h+1, \ldots, 0\},$$

where $\psi(s), \phi(s) \in \mathbb{R}^n_{0,+}$, $s \in \{-h, \ldots, 0\}$. The following lemmas are needed for our development.

Lemma 2. *The above two linear time-delay systems are nonnegative.*

Proof. The proof is obvious. □

Based on Lemma 2, we obtain the following results:

Lemma 3. (i) *If $\upsilon(s) \preceq \psi(s), \forall s \in \{-h, \ldots, 0\}$ then we have $x(t, \upsilon, \omega) \preceq z(t, \psi), \forall t \in \mathbb{N}$,*
(ii) *If $\psi_1(s) \preceq \psi_2(s), \forall s \in \{-h, \ldots, 0\}$ then we have $z(t, \psi_1) \preceq z(t, \psi_2), \forall t \in \mathbb{N}$,*
(iii) *If $\phi(s) \preceq \upsilon(s), \forall s \in \{-h, \ldots, 0\}$ then we have $g(t, \phi) \preceq x(t, \upsilon, \omega), \forall t \in \mathbb{N}$.*

Proof. (i) Denote $e(t) = z(t) - x(t)$, $\varepsilon(t) = \overline{\omega} - \omega(t)$ and consider the following system

$$e(t+1) = A_0e(t) + A_1e(t-\tau_1(t)) + B\varepsilon(t), \quad t \in \mathbb{N} \tag{5}$$
$$e(s) = \psi(s) - \upsilon(s), \quad s \in \{-h, -h+1, \ldots, 0\}.$$

By Lemma 2, we have $e(t, \psi - \upsilon, \varepsilon) \succeq 0, \forall t \in \mathbb{N}$. This implies that $x(t, \upsilon, \omega) \preceq z(t, \psi), \forall t \in \mathbb{N}$.

(ii) and (iii) Similarly, we also have (ii) and (iii). The proof of Lemma 3 is completed. □

Lemma 4 (Berman & Plemmons, 1994). *Let $M \in \mathbb{R}^{n\times n}$ be a nonnegative matrix. Then the following statements are equivalent:* (i) $\rho(M) < 1$; (ii) $(I-M)^{-1} \succeq 0$; (iii) $\exists p \succ 0, (M-I)p \prec 0$.

We are now in a position to introduce the main result in the form of the following theorem.

Theorem 5. *If $\rho(A_0 + A_1) < 1$ then*

(i) *vector $\overline{q} = (I - A_0 - A_1)^{-1}B\overline{\omega}$ is the smallest componentwise ultimate upper bound of system (1); and*
(ii) *vector $\underline{q} = (I - A_0 - A_1)^{-1}B\underline{\omega}$ is the largest componentwise ultimate lower bound of system (1).*

Proof. (i) Since $\rho(A_0 + A_1) < 1$, by Lemma 4, $\overline{q}$ exists and is nonnegative. First, we prove that, for any nonnegative initial condition $\psi(.)$, $\limsup_{t\to\infty} z(t, \psi) \preceq \overline{q}$. Indeed, for any nonnegative initial condition $\psi(s) \succeq 0$, $s \in \{-h, \ldots, 0\}$, by Lemma 4, there exists a positive vector η such that $(A_0 + A_1)\eta \preceq \eta$ and $\psi(s) \preceq \overline{q} + \eta, \forall s \in \{-h, \ldots, 0\}$. Set a function $\psi_\eta(s) = \overline{q} + \eta$, $s \in \{-h, \ldots, 0\}$, then by Lemma 3, we have

$$z(t, \psi) \preceq z(t, \psi_\eta), \quad \forall t \in \mathbb{N}. \tag{6}$$

For every $s \in \mathbb{N}$, we define the set

$$I_s = \{s(h+1) + i,\ i = 1, 2, \ldots, h+1\}. \tag{7}$$

We will prove that

$$z(t, \psi_\eta) \preceq \overline{q} + (A_0 + A_1)^s\eta, \quad \forall s \in \mathbb{N},\ \forall t \in I_s. \tag{8}$$

Indeed, for $s = 0$ and $t = 1$, by using the assumption $\rho(A_0 + A_1) < 1$ and $\overline{q} = (I - A_0 - A_1)^{-1}B\overline{\omega}$, we have

$$\begin{aligned} z(1, \psi_\eta) &= A_0z(0, \psi_\eta) + A_1z(-\tau_1(0), \psi_\eta) + B\overline{\omega} \\ &\preceq A_0(\overline{q} + \eta) + A_1(\overline{q} + \eta) + B\overline{\omega} \\ &= (A_0 + A_1)\overline{q} + B\overline{\omega} + (A_0 + A_1)\eta \\ &= \overline{q} + (A_0 + A_1)\eta \\ &\preceq \overline{q} + \eta. \end{aligned} \tag{9}$$

Similarly, we also have,

$$\begin{aligned} z(t, \psi_\eta) &\preceq \overline{q} + (A_0 + A_1)\eta \\ &\preceq \overline{q} + \eta, \quad \forall t \in \{2, \ldots, h+1\}. \end{aligned} \tag{10}$$