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Rigid body pose estimation based on the Lagrange–d'Alembert principle[☆]

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ABSTRACT

Stable estimation of rigid body pose and velocities from noisy measurements, without any knowledge of the dynamics model, is treated using the Lagrange–d'Alembert principle from variational mechanics. With body-fixed vision and inertial sensor measurements, a Lagrangian is obtained as the difference between a kinetic energy-like term that is quadratic in velocity estimation error and the sum of two artificial potential functions; one obtained from a generalization of Wahba's function for attitude estimation and another which is quadratic in the position estimate error. An additional dissipation term that is linear in the velocity estimation error is introduced, and the Lagrange–d'Alembert principle is applied to the Lagrangian with this dissipation. A Lyapunov analysis shows that the state estimation scheme so obtained provides stable asymptotic convergence of state estimates to actual states in the absence of measurement noise, with an almost global domain of attraction. This estimation scheme is discretized for computer implementation using discrete variational mechanics, as a first order Lie group variational integrator. The discrete estimation scheme can also estimate velocities from such onboard sensor measurements. Moreover, all states can be estimated during time periods when measurements of only two inertial vectors, the angular velocity vector, and one feature point position vector are available in body frame. In the presence of bounded measurement noise in the vector measurements, numerical simulations show that the estimated states converge to a bounded neighborhood of the true states.

1. Introduction

Estimation of coupled translational and rotational motion is indispensable for operations of spacecraft, unmanned aerial and underwater vehicles. Autonomous state estimation of a rigid body based on inertial vector measurement and visual feedback from stationary landmarks (Karpenko, Konovalenko, Miller, Miller, & Nikolaev, 2015; Miller & Miller, 2015), in the absence of a dynamics model for the rigid body, is analyzed here. The estimation scheme proposed here can also be applied to *relative state* estimation with respect to moving objects (Misra, Izadi, Sanyal, & Scheeres, 2015). This estimation scheme can enhance the autonomy and reliability of unmanned vehicles in uncertain GPS-denied environments. Salient features of this estimation scheme are (1) use of onboard optical and inertial sensors, with or without rate gyros, for autonomous navigation; (2) robustness to uncertainties and lack of knowledge of dynamics; (3) low computational complexity for easy implementation with onboard processors; (4) proven stability with large domain of attraction for state estimation errors; and (5) versatile enough to estimate motion with respect to stationary as well as moving objects. Robust state estimation of rigid bodies in the absence of complete knowledge of their dynamics, is required for their safe, reliable, and autonomous operations in poorly known conditions. In practice, the dynamics of a vehicle may not be perfectly known, especially when the vehicle is under the action of poorly known forces and moments. The scheme proposed here has a single, stable algorithm for the coupled translational and rotational motion of rigid bodies using onboard optical and inertial sensors. This avoids the need for measurements from external sources, like GPS, which may not be available in indoor, underwater or cluttered environments (Amelin & Miller, 2014; Leishman, McLain, & Beard, 2014; Miller & Miller, 2014).

Attitude estimators using unit quaternions for attitude representation may be *unstable in the sense of Lyapunov*, unless

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they identify antipodal quaternions with a single attitude. This is also the case for attitude control schemes based on continuous feedback of unit quaternions, as shown in Bayadi and Banavar (2014); Chaturvedi, Sanyal, and McClamroch (2011); Sanyal, Fosbury, Chaturvedi, and Bernstein (2009). One adverse consequence of these unstable estimation and control schemes is that they end up taking longer to converge compared with stable schemes under similar initial conditions and initial transient behavior. Continuous-time attitude observers and filtering schemes on SO(3) and SE(3) have been reported in, e.g., Bonnabel, Martin, and Rouchon (2009); Khosravian, Trumpf, Mahony, and Hamel (2015); Khosravian, Trumpf, Mahony, and Lageman (2015); Mahony, Hamel, and Pflimlin (2008); Maithripala, Berg, and Dayawansa (2004); Markley (2006); Rehbinder and Ghosh (2003); Sanyal, Lee, Leok, and McClamroch (2008); Vasconcelos, Cunha, Silvestre, and Oliveira (2010); Vasconcelos, Silvestre, and Oliveira (2008), including recent stochastic filtering approaches (Barrau & Bonnabel, 2015). These estimators do not suffer from kinematic singularities like estimators using coordinate descriptions of attitude, and they do not suffer from unwinding as they do not use unit quaternions. The maximum likelihood (minimum energy) filtering method of Mortensen (1968) was recently applied to attitude estimation, resulting in a nonlinear attitude estimation scheme that seeks to minimize the stored "energy" in measurement errors (Aguiar & Hespanha, 2006; Zamani, 2013; Zamani, Trumpf, & Mahony, 2013). This scheme is obtained by applying Hamilton–Jacobi–Bellman (HJB) theory (Kirk, 1971) to the state space of attitude motion (Zamani, 2013). Since the HJB equation can only be approximately solved with increasingly unwieldy expressions for higher order approximations, the resulting filter is only "near optimal" up to second order. Unlike approximate or "near optimal" filtering schemes that are not provably stable, the estimation scheme obtained here can be solved exactly and is almost globally asymptotically stable. Moreover, unlike filters based on Kalman filtering, the estimator proposed here does not presume any knowledge of the statistics of the initial state estimate or the sensor noise. Indeed, for vector measurements using optical sensors with limited field-of-view, the probability distribution of measurement noise needs to have compact support, unlike additive Gaussian noise processes that are commonly used.

The variational attitude estimator recently appeared in Izadi and Sanyal (2014); Izadi, Sanyal, Barany, and Viswanathan (2015); Izadi, Sanyal, Samiei, and Viswanathan (2015), where it was shown to be almost globally asymptotically stable. Advantages of this scheme over some commonly used competing schemes are reported in Izadi, Samiei, Sanyal, and Kumar (2015). This paper extends the variational estimation framework to coupled rotational (attitude) and translational motion, as exhibited by maneuvering vehicles like small UAVs. In such applications, designing separate state estimators for the translational and rotational motions may not be effective and may lead to poor navigation. For navigation and tracking the motion of such vehicles, the approach proposed here for robust and stable estimation of the coupled translational and rotational motion will be more effective than de-coupled estimation of translational and rotational motion states. Moreover, like other vision-inertial navigation schemes (Shen, Mulgaonkar, Michael, & Kumar, 2013; Shen, Mulgaonkar, Michael, & Kumar, 2013), the estimation scheme proposed here does not rely on GPS. However, unlike many other vision-inertial estimation schemes, the estimation scheme proposed here can be implemented without any direct velocity measurements. Since rate gyros are usually corrupted by high noise content and bias (Goodarzi, Lee, & Lee, 2013), such a velocity measurement-free scheme can result in fault tolerance in the case of faults with rate gyros. Additionally, this estimation scheme can be extended to relative pose estimation between vehicles from optical measurements, without direct communications or measurements of relative velocities (Misra et al., 2015).

The contents of this article are organized as follows. In Section 2, the problem of motion estimation of a rigid body using onboard optical and inertial sensors and the measurement model is introduced. The rigid body states are related to these measurements. Section 3 introduces artificial energy terms representing the measurement residuals corresponding to the rigid body state estimates. The Lagrange–d'Alembert principle is applied to the Lagrangian constructed from these energy terms with a Rayleigh dissipation term linear in the velocity measurement residual, to give the continuous time state estimator. It is shown that, in the absence of measurement noise, state estimates converge to actual states with asymptotic stability, and the domain of attraction is an open dense subset of the state space. Section 4 provides particular versions of this estimation scheme for the cases when direct velocity measurements are not available and when only angular velocity is directly measured. In Section 5, the variational pose estimator is discretized as a Lie group variational integrator, by applying the discrete Lagrange–d'Alembert principle to discretizations of the Lagrangian and the dissipation term. This estimator is simulated numerically in Section 6, for two cases: the case where at least three beacons are measured at each time instant; and the under-determined case, where occasionally less than three beacons are observed. For these simulations, true states of an aerial vehicle are generated using a given dynamics model. Optical/inertial measurements are generated, assuming bounded noise in sensor readings. Using these measurements, state estimates are shown to converge to a neighborhood of actual states, for both cases simulated. Finally, Section 7 lists the contributions and possible future extensions of the work presented in this paper.

2. Navigation using optical and inertial sensors

Consider a rigid body in spatial (rotational and translational) motion. Onboard estimation of the pose involves assigning a coordinate frame fixed to the vehicle body, and another coordinate frame fixed in space that serves as the inertial frame. Let O denote the observed environment and S denote the body. Let S denote a coordinate frame fixed to S and O be a coordinate frame fixed to O, as shown in Fig. 1. Let $R \in \mathrm{SO}(3)$ denote the rotation matrix from frame S to frame O and b denote the position of origin of S expressed in frame O. The pose (transformation) from body fixed frame S to inertial frame O is then given by

$$\mathsf{g} = \begin{bmatrix} R & b \\ 0 & 1 \end{bmatrix} \in \mathrm{SE}(3). \quad (1)$$

Consider vectors known in the inertial frame O and measured by inertial sensors in the vehicle-fixed frame S; let β be the number of such vectors. In addition, consider position vectors of a few stationary points in the inertial frame O measured by optical sensors in the vehicle-fixed frame S. Velocities of the vehicle may be directly measured or can be estimated by linear filtering of the optical position vector measurements (Izadi et al., 2015). Assume that these optical measurements are available for j points at time t, whose positions are known in frame O as p_j, $j \in \mathcal{I}(t)$, where $\mathcal{I}(t)$ denotes the index set of beacons observed at time t. Note that the observed stationary beacons or landmarks may vary over time due to the vehicle's motion. These points generate $\binom{j}{2}$ unique relative position vectors, which are the vectors connecting any two of these landmarks. When two or more position vectors are optically measured, the number of vector measurements that can be used to estimate attitude is $\binom{j}{2} + \beta$. This number needs to be