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Identification of unknown temporal and spatial load distributions in a vibrating Euler–Bernoulli beam from Dirichlet boundary measured data[☆]

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ABSTRACT

Source identification problems in a system governed by Euler–Bernoulli beam equation $\rho(x)u_{tt} + (r(x)u_{xx})_{xx} = F(x)H(t)$, $(x, t) \in (0, l) \times (0, T)$, from available boundary observation (measured data), namely, from measured slope $\theta(t) := u_x(0, t)$ at $x = 0$, are considered. We propose a new approach to identifying the unknown temporal ($H(t)$) and spatial ($F(x)$) loads. This novel approach is based on weak solution theory for PDEs and quasi-solution method for inverse problems combined with the adjoint method. It allows to construct not only a mathematical theory of inverse source problems for Euler–Bernoulli beam, but also an effective numerical algorithm for reconstruction of unknown loads. Introducing the input–output operators $(\Phi H)(t) := u_x(0, t; H)$ and $(\Psi F)(t) := u_x(0, t; F)$, $t \in (0, T)$, we show that both operators are compact. Based on this result and general regularization theory, we prove an existence of unique solutions of the regularized normal equations $(\Phi^*\Phi + \alpha I)H_\alpha = \Phi^*\theta$ and $(\Psi^*\Psi + \alpha I)F_\alpha = \Psi^*\theta$. Then we develop the adjoint problem approach to prove Fréchet differentiability of the corresponding cost functionals and Lipschitz continuity of the Fréchet gradients. Derived explicit gradient formulas via the adjoint problem solution and known load, allow use of gradient type convergent iterative algorithms. Results of numerical simulations for benchmark problems illustrate robustness and high accuracy of the algorithm based on the proposed approach.

1. Introduction

The beam is one of the fundamental elements of an engineering structure. Vibration problems related to the static and dynamic response of beams have huge applications in building, mechanical and aircraft engineering, in earth sciences and engineering. The vibration problems have been studied since end of the 18th century, beginning from the work of Aitken (1878). We refer to Gladwell (2004) and Morassi (2007) and references therein. For other engineering applications, including use of waveguides with different mechanical and geometric properties and also an adaptive control strategy to control the performance of tall buildings under seismic loads, we refer to Bartoli, Marzani, di Scalea, and Viola (2006), Hurlebaus and Gaul (2006) and Mi, Michaels, and Michaels (2006) and references therein.

The most commonly used beam models are based on the classical Euler–Bernoulli beam theory, which is regarded as the basic model in all the above mentioned scientific fields. For non-homogeneous dynamic Euler–Bernoulli beam, vibration is governed by the following equation: $\rho(x)u_{tt} + (r(x)u_{xx})_{xx} = F(x)H(t)$. Here $u(x, t)$ is the displacement function, depending on the space $x \in (0, l)$ and time $t \in (0, T)$ variables, $F(x)$ and $H(t)$ are the spatial and temporal load distributions. Further, $r(x) = EI(x)$, $E > 0$ is the elasticity modulus, $I(x) > 0$ is the moment of inertia of the cross-section, $\rho(x)$ is the mass density of the beam. In control theory of PDEs, the parameter identifiability for distributed parameter systems governed by Euler–Bernoulli equation has also attracted a great deal of attention Chang and Guo (2007), Guo (2002), Krstic, Guo, Balogh, and Smyshlyaev (2008), Krstic and Smyshlyaev (2008), Lagnese (1991) and Liu (2012). In this theory, the determination of physical parameters based on additional boundary observation (or measured data) is referred to as an identification problem. Since parameter identification problems are a

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subclass of inverse problems, these problems are studied also in inverse problems theory and applications. From the point of view of methods/approaches used, the studies on the parameter identifiability for Euler–Bernoulli equation can be broadly divided into two categories: methods based on spectral theory and methods based on observations, i.e. input–output mappings. In the pioneering studies these problems have been studied using the first category of methods, as inverse spectral problems (Barcilon, 1986; Gladwell, 1986; McLaughlin, 1984). The main effort in all these works was determination of physical parameters of beams from the spectral data. However, in practice, it is difficult to acquire spectral data, as a measured output data.

An identification of variable spatial coefficients in the Euler–Bernoulli equation from boundary input and observations, i.e. based on the second category of methods, has been considered in Chang and Guo (2007), Lesnic and Hasanov (2008) and Lesnic, Elliott, and Ingham (1999). In particular, it is proved in Chang and Guo (2007) that the unknown coefficients $\rho(x)$ and $r(x)$ can be uniquely determined by the given boundary input $g(t) := ((r(x)u_{xx}(x,t))_x)_{x=l}$ and the boundary observations $u(l,t)$ and $u_x(l,t)$, available for all $t \in (0,T)$. More precisely, an equivalence of two identification problems: the problem of identifying the unknown coefficients $\rho(x)$ and $r(x)$ from the spectral data $\{\omega_n, \phi_n(l), \phi_n'(l)\}_{n=1}^{\infty}$ and the problem of identifying these coefficients from the boundary measured (input–output) data $g(t) \mapsto u(l,t)$ and $g(t) \mapsto u_x(l,t)$, is proved. On one hand this fundamental result is to bridge a gap between two above mentioned category of methods. On the other hand, this result shows that it is possible to determine uniquely the unknown coefficients from most feasible and available boundary observations.

While the study of coefficient identification problems is enough comprehensive, to our best knowledge, only few results are known for source identification problems related to Euler–Bernoulli equation with boundary/final observations (Hasanov, 2009; Hasanov & Baysal, 2015; Kawano, 2014; Liu, 2012; Nicaise & Zair, 2004). Specifically, two types of boundary observations are used in Nicaise and Zair (2004) for space-wise dependent source identification problems related to the *constant coefficient* dynamic Euler–Bernoulli equation $u_{tt} + u_{xxxx} = \lambda(t)a(x)$, with special form of the unknown spatial load $a(x) = \sum_{k=1}^{K} \alpha_k \delta(x - \xi_k)$ and with given smooth temporal load $\lambda \in C^1([0,T])$. In the first identification problem, the authors proved that the unknown spatial load $a(x)$ can be uniquely determined from the boundary observation $u_{xx}(0,t)$, $t \in (0,T)$, i.e. measured value of curvature. In the second identification problem, taking the hinged–hinged boundary conditions $u(x,t) = u_{xx}(x,t) = 0, x \in \{0,l\}$, in the forward (direct) problem, it is proved that the unknown spatial load can be determined uniquely from the boundary observation $u_x(0,t)$, $t \in (0,T)$, i.e. measured value of slope at $x = 0$. This source identification problem has been then considered in Kawano (2014) for more general Euler–Bernoulli equation $w_{tt} + (\mu/\rho)w_t + (EI/\rho)u_{xxxx} - (T_{tr}/\rho)u_{xx} = h(x,t)$, which includes the damping coefficient $\mu \geq 0$ and the traction force $T_{tr} \geq 0$ along the beam. Moreover, it is assumed that the loading is asynchronous and has the form $h(x,t) = \sum_{n=1}^{K} g_n(t)f_n(x)$, where $f_n \in H^{-2}$, which permits one to consider also continuous beams with $K > 1$ internal supports. Different from Nicaise and Zair (2004), in Kawano (2014) more specific, *interior* observations are used as a measured output data for determination of the unknown functions $f_n(x)$, $n = \overline{1,K}$. An effective combination of the Lie-group adaptive method and the differential quadrature method is proposed in Liu (2012) for numerical recovering an unknown space and time dependent load in a constant coefficient Euler–Bernoulli beam equation.

For the variable coefficient Euler–Bernoulli equation $\rho(x)u_{tt} + (r(x)u_{xx})_{xx} = F(x,t)$, identification problems with final observations $u_T(x) := u(x,T)$ (final displacement) and $\nu_T(x) := u_t(x,T)$ (final speed) has been formulated in Hasanov (2009). Using the least squares (or quasi-solution) method combined with the adjoint problem approach, here explicit formulas for the Fréchet derivatives of the cost functionals are derived via the solutions of the corresponding adjoint problems. Then the Lipschitz continuity of the gradients is proved and sufficient conditions for uniqueness of inverse problems solutions are derived. The theory developed here is then applied in Hasanov and Baysal (2015) to the problem of determining the unknown spatial load $F(x)$ from the final displacement observation in a cantilever beam governed by the equation $m(x)u_{tt} + (EI(x)u_{xx})_{xx} = F(x)H(t)$.

Spatial load distribution identification problems have been studied in Hasanov (2009), Hasanov and Baysal (2015), Kawano (2014) and Nicaise and Zair (2004), assuming that the temporal source $H(t)$ is a known function. Of course, there is no doubt that these type of identification problems arise in many different engineering fields, as it is remarked above. However, to the best knowledge of authors, it is the temporal distribution of the load that is more difficult and important to be studied in the first place. In many systems, especially in the case of a cantilever beam, the spatial load distribution can even be recognized by simple inspections. This is a specific motivation for the interest in inverse source problems related to identification of temporal load distribution from available boundary observation.

In the presented work we study two typical and important, from an engineering application point of view, source identification problems (SIPs): the problem of identifying the temporal load distribution $H(t)$ and the problem of identifying the spatial load distribution $F(x)$ in a vibrating beam, from boundary observation $\theta(t) := u_x(0,t)$, i.e. slope at $x = 0$. The dynamic vibration of an Euler–Bernoulli beam is governed by the following forward problem:

$$\begin{cases} \rho(x)u_{tt} + (r(x)u_{xx})_{xx} = F(x)H(t), & (x,t) \in \Omega_T, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in (0,l), \\ u(0,t) = u_{xx}(0,t) = u(l,t) = u_x(l,t) = 0, & t \in (0,T), \end{cases} \tag{1}$$

where $\Omega_T := (0,l) \times (0,T)$. This initial–boundary value problem, with the mixed boundary conditions, describes many engineering models, in particular, the most commonly used hinged–clamped bridge model (Han, Benarova, & Wei, 1999).

In the first source identification problem (subsequently, SIP(H)), the spatial source $F \in L^2(0,l)$ is the input and the temporal source $H \in L^2(0,T)$ is unknown and needs to be identified from the boundary observation

$$\theta(t) := u_x(0,t), \quad t \in (0,T]. \tag{2}$$

In the second identification problem (subsequently, SIP(F)), one needs to identify the unknown spatial source $F(x)$ in (1), from the boundary observation (2), assuming that the temporal source $H(t)$ is a given input. Our approach is based on weak solution theory of PDEs applied to the direct problem (1), which allows use of non-smooth input as well as noisy output data, and the quasi-solution method for inverse problems, which choice is motivated by the following fact. Let us introduce the input–output mapping

$$(\Phi H)(t) := (u_x(x,t;H))_{x=0}, \tag{3}$$

where $u(x,t;H)$ is the weak solution of the forward problem (1), corresponding to a given (admissible) source $H \in L^2(0,T)$. Then SIP(H) can be reformulated in the operator equation form as $\Phi H = \theta$. By the same way, the input–output mapping

$$(\Psi F)(t) := (u_x(x,t;F))_{x=0}, \tag{4}$$

corresponding to SIP(F) can be introduced; here $u(x,t;F)$ is the weak solution of the forward problem (1) for a given (admissible) source $F \in L^2(0,l)$. In this case SIP(F) can be reformulated as the operator equation $\Psi F = \theta$.