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Brief paper

Multi-player pursuit–evasion games with one superior evader☆

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ABSTRACT

Inspired by the hunting and foraging behaviors of group predators, this paper addresses a class of multi-player pursuit–evasion games with one superior evader, who moves faster than the pursuers. We are concerned with the conditions under which the pursuers can capture the evader, involving the minimum number and initial spatial distribution required as well as the cooperative strategies of the pursuers. We present some necessary or sufficient conditions to regularize the encirclement formed by the pursuers to the evader. Then we provide a cooperative scheme for the pursuers to maintain and shrink the encirclement until the evader is captured. Finally, we give some examples to illustrate the theoretical results.

1. Introduction

There was an interesting Chinese proverb saying: "If the tiger went down to plain land, it would be insulted by dogs", which means that even though the tiger has better explosive force and a faster speed, it might be insulted or hunted by canids (hyenas) who have slower speed but are adept in besieging their preys cooperatively. In the real world, this hunting or foraging behavior by predators for a larger or faster prey is a wide spread phenomenon (Packer & Ruttan, 1988; Vicsek, 2010). For example, the pronghorn's speed is usually 80–100 km/h while the lion's speed is only 70–80 km/h. But a group of lions are able to capture the pronghorn through effective cooperation.

These hunting phenomena can be naturally generalized to the field of robotics and control, where multiple slower robots (pursuers) try to capture one faster target (evader) who, conversely, attempts to escape. Theoretically, it is known as multi-player pursuit–evasion games with one superior evader (Wang, Cruz, Chen, Pham, & Blasch, 2007). Here, the term "superior" signifies that the evader has comparatively more advantageous control resources than the pursuers.

Pursuit–evasion game, as a common model in differential game theory, has been studied by many researchers during the past decades (Bhattacharya & Başar, 2013; Bopardikar, Smith, & Bullo, 2011; Eklund, Sprinkle, & Sastry, 2012; Shen, Pham, Blasch, Chen, & Chen, 2011), involving extensive applications such as interception problems of missile and satellite, formation control and jamming confrontation of unmanned aerial vehicles (UAVs), search and rescue operations of robots, and so on. In recent years, the group behaviors (such as team collaboration, distributed control, artificial intelligence, population evolution, etc.) in multi-agent systems have received increasing attention (Bauso, Tembine, & Başar, 2015; Cai & Huang, 2016; Chen, Gan, Huang, Dou, & Fang, 2016; Semsar-Kazerooni & Khorasani, 2009; Vamvoudakis, Lewis, & Hudas, 2012). The conventional approach introduced by Isaacs (1965) is applicable to two-player pursuit–evasion game, which is based on the underlying idea of state reversal: Starting from the terminal manifold, an optimal trajectory of the states is depicted retrograde and the value function of the game can be determined by using a formulation of the Hamilton–Jacobi–Isaacs (HJI) equation, so as to attain the equilibrium strategies for the players. However, when it comes to the case of multiple players, this approach encounters tremendous difficulties in the determination of terminal manifold, the characterization of cooperation among multiple pursuers or evaders, and the computational complexity in solving HJI equations.

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To cope with the above challenges, researchers generally demonstrate the possibility of capture or escape by exhibiting some particular strategies or policies of the players (Bakolas & Tsiotras, 2012; Bhadauria, Klein, Isler, & Suri, 2012; Bopardikar, Bullo, & Hespanha, 2008, 2009; Lin, Qu, & Simaan, 2011; Zha, Chen, Peng, & Gu, in press). These methods can be collectively referred to as "method of explicit policy" (Isaacs, 1965; Zha et al., in press). For example, Bopardikar et al. (2008) proposed a sweep–pursuit–capture strategy to capture a single evader for multiple pursuers in an unbounded planar environment and determined the minimum probability of capture. In Bopardikar et al. (2009) they addressed a cooperative Homicidal Chauffeur game and presented a multi-phase cooperative strategy (align, swerve, encircle and close phases). Lin et al. (2011) designed entrapment strategies for the pursuers under distributed information (sensing limitation) that the pursuers will spread out around the evader while approaching it.

However, most of the literature assumes that the pursuers move faster than or equal to the evader to ensure capture feasible. Little has been done for the problems with faster evader in a continuous unbounded planar environment as the hunting phenomena in the animal kingdom mentioned before. Breakwell might be the first one to consider the multi-player pursuit–evasion problems with faster evader in the literature (Breakwell, 1974; Hagedorn & Breakwell, 1976), where the evader is required to pass between two pursuers. He obtained a closed-form solution by dividing the optimal trajectories of the players into two successive phases, but it is not scalable for more pursuers. Wang et al. (2007) preliminarily studied the multi-pursuer single-superior-evader game on an unbounded plane and provided the minimum number of the pursuers required when the evader moves in a straight line.

Other literature related to faster evader usually imposes some additional constraints on the problem formulation, such as polygonal environment (a closed and bounded set in Euclidean space with polygonal boundary) (Stiffler & O'Kane, 2014), graph environment (discrete time and discrete space with multiple nodes and edges) (Vieira, Govindan, & Sukhatme, 2009), sensing limitation (Lin, Qu, & Simaan, 2013) and limited turning of the evader (Exarchos, Tsiotras, & Pachter, 2015). With regard to a general planar environment where the motions of the players are simple, however, some fundamental problems have not been solved completely. For example, how many pursuers would be necessary to capture an evader? What conditions make the capture or escape possible? How to design the cooperative scheme among the pursuers?

The main contribution of this paper is to solve the above problems. We obtain the minimum number of pursuers required to guarantee a capture, which only depends on the speed ratio of the pursuers to the evader. Then we present some necessary or sufficient conditions to regularize the encirclement formed by the pursuers to the evader, and provide a cooperative scheme for the pursuers to maintain and shrink the encirclement until the evader is captured. This cooperative scheme contains three phases: besiege, shrink, and capture. For each phase, the feasible strategies of the players are characterized.

In our previous work (Zha et al., in press), a fishing game is introduced, which bears resemblance to the pursuit game in Hagedorn and Breakwell (1976), i.e., the superior evader must pass the gap between two pursuers. We obtained a complete solution which can be utilized to induce the analysis of capture conditions for the current multi-player game, because the evader will exert itself to break through the encirclement against some two adjacent pursuers on each instant of time. Thus in Section 2, we present the problem formulation based on the work on fishing game. Then in Section 3, we focus on determining the capture conditions and cooperative strategies for the pursuers. In Section 4, we give some examples to illustrate our findings. Finally, conclusions and future work are summarized in Section 5.

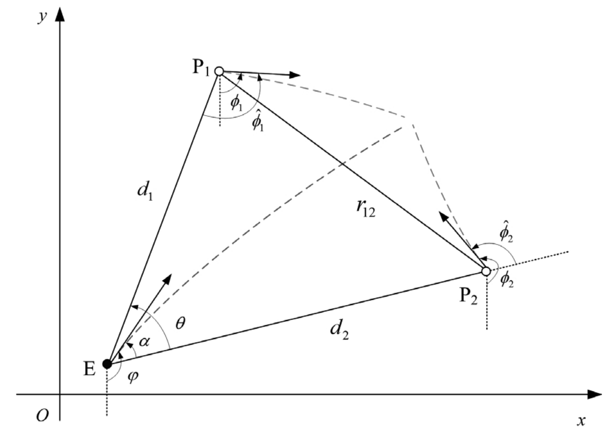

Fig. 1. Fishing game in the fixed reference system.

2. Preliminaries

2.1. A fishing game with two pursuers and one evader

First, we provide a brief introduction to our previous work on the fishing game (Zha et al., in press), which is played in the plane by two pursuers and one superior evader. The evader attempts to pass the gap between the two pursuers (as shown in Fig. 1), while avoiding being captured. On the contrary, the pursuers try to capture the evader while preventing the evader from threading.

In a reduced state space formed by the relative distances d_1, d_2 and the included angle θ, the dynamics are given as follows:

$$\dot{d}_1 = -v_p \cos\hat{\phi}_1 - v_e \cos(\theta - \alpha), \qquad d_1(t_0) = d_1^0 \quad (1)$$

$$\dot{d}_2 = v_p \cos\hat{\phi}_2 - v_e \cos\alpha, \qquad d_2(t_0) = d_2^0 \quad (2)$$

$$\dot{\theta} = -\frac{v_p}{d_1}\sin\hat{\phi}_1 + \frac{v_e}{d_1}\sin(\theta - \alpha) - \frac{v_p}{d_2}\sin\hat{\phi}_2 + \frac{v_e}{d_2}\sin\alpha, \qquad \theta(t_0) = \theta_0. \quad (3)$$

The fishing game with point capture terminates when one of the following situations occurs: (a) $\min\{d_1, d_2\} = 0$, the pursuers win; (b) $\min\{d_1, d_2\} > 0$ and $\theta = \pi$, the evader wins. Then we obtained a complete solution that consists of a barrier and the optimal strategies of the players. Here, we give some main results.

Theorem 2.1. *The barrier of fishing game is governed by*

$$\mathscr{B} = \left\{d_1, d_2, \theta \mid \cos\theta = \frac{(1-a^2)(d_1+d_2)^2}{2d_1 d_2} - 1\right\} = \{d_1, d_2, \theta \mid r_{12} = a(d_1+d_2)\} \quad (4)$$

and the optimal strategies of the players on the barrier are given by

$$\hat{\phi}_1^* = \frac{\pi}{2}, \qquad \hat{\phi}_2^* = \frac{\pi}{2} \quad (5)$$

$$\sin\alpha^* = \frac{d_1 - d_2\cos\theta}{r_{12}}, \qquad \cos\alpha^* = \frac{d_2\sin\theta}{r_{12}} \quad (6)$$

where $r_{12} = \sqrt{d_1^2 + d_2^2 - 2d_1 d_2\cos\theta}$ is the distance between the pursuers P_1 and P_2, and $a = v_p/v_e < 1$.

The barrier separates the state space into two disjoint regions: capture zone and escape zone. From Theorem 2.1, the capture zone can be described by

$$\mathscr{D}_p = \left\{d_1, d_2, \theta \mid \cos\theta \geq \frac{(1-a^2)(d_1+d_2)^2}{2d_1 d_2} - 1\right\} = \{d_1, d_2, \theta \mid r_{12} \leq a(d_1+d_2)\} \quad (7)$$