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Brief paper

# Dynamic output feedback $\mathcal{H}_\infty$ control of continuous-time switched affine systems[☆]

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ABSTRACT

This paper deals with output feedback switching function control design for continuous-time switched affine systems, assuring global asymptotic stability of a desired equilibrium point. The set of all attainable equilibrium points is provided. More specifically, the main purpose is to design a full order switched affine filter together with a switching rule assuring not only global asymptotic stability, but also a guaranteed $\mathcal{H}_\infty$ performance level. The design conditions do not require that the open-loop subsystems be stable and outperform other techniques available in the literature to date. The results are compared with recent ones based on a quadratic Lyapunov function and on a max-type Lyapunov function. Numerical examples illustrate the theoretical results and are used for comparisons.



## 1. Introduction

Switched systems represent an important subclass of hybrid systems composed by a finite number of subsystems and a strategy that orchestrates the switching among them. This strategy is represented by a switching rule (or function) that can be arbitrary or a control variable to be designed in order to ensure stability and guaranteed performance for the overall system. The Refs. DeCarlo, Branicky, Pettersson, and Lennartson (2000), Liberzon (2003), Shorten, Wirth, Mason, Wulff, and King (2007) and Sun and Ge (2005) are surveys on this theme. In fact, an adequate switching function assures global asymptotic stability even if all subsystems are unstable, see Liberzon (2003) and Sun and Ge (2005). In the case where the switching function is strictly consistent, it enhances the $\mathcal{H}_2$ or the $\mathcal{H}_\infty$ performance compared to the one of each isolated subsystem, see Geromel, Deaecto, and Daafouz (2013) for a discussion on this topic. Due to their intrinsic characteristics, the switched systems are of great interest in practice, mainly in the area of power electronics, see Cardim, Teixeira, Assunção, and Covacic (2009), Corona, Buisson, De Schutter, and Giua (2007), Deaecto, Geromel, Garcia, and Pomílio (2010) and Garcia, Pomílio, Deaecto, and Geromel (2009).

Several systems of this class can be described as switched affine systems. They are characterized by presenting affine terms allowing the switched system to have several equilibrium points that constitute a region of great interest in the state space. Clearly, the switched linear system is a particular case, characterized by the fact that all affine terms are null and, consequently, the unique equilibrium point is the origin. The control design problem for switched affine systems is more involved, since it consists in two main goals: find a region of attainable equilibrium points and design a switching function responsible to guide any trajectory of the closed-loop system to the desired equilibrium. The literature provides important results for the case where the state is available for feedback. Specifically, Refs. Bolzern and Spinelli (2004), Deaecto et al. (2010), Hetel and Fridman (2013), Kuiava, Ramos, Pota, and Alberto (2013) and Xu, Zhai, and He (2008) are based on a quadratic Lyapunov function, Refs. Scharlau, De Oliveira, Trofino, and Dezuo (2014) and Trofino, Scharlau, Dezuo, and De Oliveira (2012) are based on a max-type Lyapunov function and Corona et al. (2007), Hauroigne, Riedinger, and Iung (2011) and Seatzu, Corona, Giua, and Bemporad (2006) are based on optimal control. Some of them, as for instance, Hauroigne et al. (2011), Hetel and Fridman (2013), Kuiava et al. (2013) and Xu et al. (2008) do not assure global asymptotic stability, but practical stability, where the state is not guided to an equilibrium point, but to a region near the desired equilibrium. Regarding performance, Ref. Deaecto et al. (2010) proposes a switching function assuring a guaranteed cost level, and Ref. Trofino et al. (2012) deals with $\mathcal{H}_\infty$ performance. For output feedback control design of switched affine systems, the literature provides only a

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few results regarding the design of a stabilizing switching function, as for instance, Ref. Yoshimura, Assunção, Da Silva, Teixeira, and Mainardi Jr. (2013) provides a switched observer-based solution. Ref. Grala and Trofino (2014) treats switched observers design for state and uncertain parameter estimation of uncertain affine systems. For switched linear systems, Refs. Deaecto, Geromel, and Daafouz (2011) and Geromel, Colaneri, and Bolzern (2008) provide an output feedback switching function assuring $\mathcal{H}_\infty$ and $\mathcal{H}_2$ guaranteed performances, respectively.

This paper proposes an output feedback switching function for continuous-time switched affine systems, preserving global asymptotic stability of an equilibrium point, which belongs to a set of attainable ones. Actually, our main contribution is to determine a set of full order switched affine filters together with a switching function assuring an $\mathcal{H}_\infty$ guaranteed performance. Our design procedure is based on necessary and sufficient conditions for the existence of a structured solution for the inequalities provided in Deaecto and Santos (2015), which assure stability and guaranteed performance in the state feedback case. The technique does not require that the open-loop subsystems be stable. Moreover, the same property pointed out in Geromel et al. (2008) for switched linear filters is also valid here, that is, considering only stability, we show that the proposed switched affine filter presents an observer-based structure. This paper introduces a separation property that is useful for the solution of the design conditions. Our technique is compared theoretically with the one proposed in Yoshimura et al. (2013) and numerically with the technique based on a max-type Lyapunov function. Illustrative examples show the validity of the proposed theory.

The notation is standard. For real matrices or vectors ($'$) indicates transpose. For symmetric matrices, the symbol ($\bullet$) denotes each of its symmetric blocks, $I$ denotes the identity matrix and $\mathrm{Tr}(\cdot)$ denotes the trace function. The convex combination of matrices with the same dimensions $\{J_1, \dots, J_N\}$ is denoted by $J_\lambda = \sum_{j=1}^{N} \lambda_j J_j$ where $\lambda$ belongs to the unitary simplex $\Lambda$ composed by all nonnegative vectors $\lambda \in \mathbb{R}^N$ such that $\sum_{j=1}^{N} \lambda_j = 1$. The squared norm of a trajectory $\xi(t)$ defined for all $t \geq 0$, denoted by $\|\xi\|_2^2$ is equal to $\|\xi\|_2^2 = \int_0^\infty \xi(t)'\xi(t)dt$. The set of all finite norm trajectories, such that $\|\xi\|_2^2 < \infty$ is denoted by $\mathcal{L}_2$. The Hermitian of a real matrix $M$ is defined as $\mathrm{He}\{M\} = M + M'$. The set composed by the first $N$ positive integers is denoted by $\mathbb{K} = \{1, \dots, N\}$. A square real matrix is said to be Hurwitz stable if its eigenvalues have strictly negative real parts.

## 2. Problem statement

Consider a continuous-time switched affine system with state space realization

$$\dot{x}(t) = A_\sigma x(t) + H_\sigma w(t) + b_\sigma \tag{1}$$
$$y(t) = C_\sigma x(t) + D_\sigma w(t) \tag{2}$$
$$z(t) = E_\sigma x(t) \tag{3}$$

evolving from the initial condition $x(0) = x_0 \in \mathbb{R}^{n_x}$, where $x(t) \in \mathbb{R}^{n_x}$ is the state, $w(t) \in \mathbb{R}^{n_w}$ is the external input, $y(t) \in \mathbb{R}^{n_y}$ is the measured output and $z(t) \in \mathbb{R}^{n_z}$ is the controlled output. The switching function $\sigma(t) : t \geq 0 \to \mathbb{K}$ selects at each instant of time one of the $N$ available subsystems. Note that, whenever $b_i \neq 0$ the switched system presents several equilibrium points $x_e \in X_e \subset \mathbb{R}^{n_x}$ belonging to a set of attainable ones defined by

$$X_e = \{x_e \in \mathbb{R}^{n_x} \; : \; x_e = -A_\lambda^{-1} b_\lambda, \; A_\lambda \in \mathcal{H}, \; \lambda \in \Lambda\} \tag{4}$$

with $\mathcal{H}$ being the set of all Hurwitz stable matrices. For a relatively small number of subsystems this set can be determined by line search with respect to the components of $\lambda \in \Lambda$. Unfortunately, in the general case, for an arbitrary number of subsystems, this task may not be trivial since it involves the solution of a NP-hard problem, see Blondel and Tsitsiklis (1997).

Our main goal is to design a switching function $\sigma(y) : \mathbb{R}^{n_y} \to \mathbb{K}$ preserving global asymptotic stability of a desired equilibrium point $x_e \in X_e$, which is selected from the choice of an adequate $\lambda(x_e) \in \Lambda$. To this end, we propose to design a full order switched affine filter with state space realization

$$\dot{\hat{x}}(t) = \hat{A}_\sigma \hat{x}(t) + \hat{B}_\sigma y(t) + \hat{b}_\sigma \tag{5}$$

evolving from the initial condition $\hat{x}(0) = \hat{x}_0$, where $\hat{x}(t) \in \mathbb{R}^{n_x}$ is the filter state and the matrices $(\hat{A}_i, \hat{B}_i, \hat{b}_i), \; \forall i \in \mathbb{K}$, of compatible dimensions have to be determined. Using the measured output $y(t)$, the filter will provide important information to implement the proposed switching function.

Setting the initial condition $x_0 = x_e$, we can define the following $\mathcal{H}_\infty$ performance index associated with an arbitrary stabilizing switching function $\sigma(\cdot)$, namely

$$J_\infty(\sigma) = \sup_{0 \neq w \in \mathcal{L}_2} \frac{\|z_e\|_2^2}{\|w\|_2^2} \tag{6}$$

where $z_e(t) = E_\sigma(x(t) - x_e)$. Actually, this performance index has already been introduced in Deaecto and Santos (2015). The rationale behind this definition is that for a fixed switching rule $\sigma(t) = i, \; \forall t \geq 0$, this index reduces to the $\mathcal{H}_\infty$ squared norm of the transfer function from the input $w$ to the controlled output $z_e$ of the $i$th subsystem

$$\dot{\xi}(t) = A_i \xi(t) + H_i w(t), \quad \xi(0) = 0 \tag{7}$$
$$z_e(t) = E_i \xi(t) \tag{8}$$

with $\xi(t) = x(t) - x_e$, whenever it is asymptotically stable. This system was obtained from (1)–(2) imposing the fixed switching rule $\sigma(t) = i \in \mathbb{K}, \; \forall t \geq 0$, for which the unique equilibrium point is $x_e = -A_i^{-1} b_i \in X_e$.

Considering $\hat{x}(0) = x_e$, connecting the switched affine filter (5) to the system (1)–(3) and defining the augmented state vector $\tilde{\xi} = [(x - x_e)' \; (\hat{x} - x_e)']'$, we obtain

$$\dot{\tilde{\xi}}(t) = \tilde{A}_\sigma \tilde{\xi}(t) + \tilde{H}_\sigma w(t) + \tilde{\ell}_\sigma \tag{9}$$
$$z_e(t) = \tilde{E}_\sigma \tilde{\xi}(t) \tag{10}$$

where $\tilde{\xi}(0) = 0$ and the indicated matrices are such that

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ \hat{B}_i C_i & \hat{A}_i \end{bmatrix}, \qquad \tilde{H}_i = \begin{bmatrix} H_i \\ \hat{B}_i D_i \end{bmatrix}, \qquad \tilde{\ell}_i = \begin{bmatrix} A_i x_e + b_i \\ (\hat{A}_i + \hat{B}_i C_i) x_e + \hat{b}_i \end{bmatrix} \tag{11}$$

and $\tilde{E}_i = [E_i \;\; 0]$. Adopting a quadratic Lyapunov function $v(\tilde{\xi}) = \tilde{\xi}' \tilde{P} \tilde{\xi}$, it will be clear in the sequel that the min-type switching function

$$\sigma(\tilde{\xi}) = \arg\min_{i \in \mathbb{K}} \tilde{\xi}' \left( -\tilde{Q}_i \tilde{\xi} + 2\tilde{P}\tilde{\ell}_i \right) \tag{12}$$

assures global asymptotic stability of the equilibrium point $x_e \in X_e$, where $\tilde{P} > 0$ and $\tilde{Q}_i, \forall i \in \mathbb{K}$, are symmetric matrices that satisfy some conditions to guarantee a certain $\mathcal{H}_\infty$ performance level. The main difficulty, in our context, is that this function cannot be directly adopted, since the system state $x \in \mathbb{R}^{n_x}$ that appears in the first component of $\tilde{\xi}$ is not available. For the moment, we need to introduce the following result from Deaecto and Santos (2015) concerning $\mathcal{H}_\infty$ state feedback control design.